

Table of Fourier Transform Pairs

Function, f(t)	Fourier Transform, F(ω)
<i>Definition of Inverse Fourier Transform</i> $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$	<i>Definition of Fourier Transform</i> $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$
$f(t - t_0)$	$F(\omega) e^{-j\omega t_0}$
$f(t) e^{j\omega_0 t}$	$F(\omega - \omega_0)$
$f(\alpha t)$	$\frac{1}{ \alpha } F\left(\frac{\omega}{\alpha}\right)$
$F(t)$	$2\pi f(-\omega)$
$\frac{d^n f(t)}{dt^n}$	$(j\omega)^n F(\omega)$
$(-jt)^n f(t)$	$\frac{d^n F(\omega)}{d\omega^n}$
$\int_{-\infty}^t f(\tau) d\tau$	$\frac{F(\omega)}{j\omega} + \pi F(0) \delta(\omega)$
$\delta(t)$	1
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$
$\text{sgn}(t)$	$\frac{2}{j\omega}$

Fourier Transform Table

UBC M267 Resources for 2005

$F(t)$	$\widehat{F}(\omega)$	Notes	(0)
$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt$	Definition.	(1)
$\frac{1}{2\pi} \int_{-\infty}^{\infty} \widehat{f}(\omega)e^{i\omega t} d\omega$	$\widehat{f}(\omega)$	Inversion formula.	(2)
$\widehat{f}(-t)$	$2\pi f(\omega)$	Duality property.	(3)
$e^{-at}u(t)$	$\frac{1}{a + i\omega}$	a constant, $\Re(a) > 0$	(4)
$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$	a constant, $\Re(a) > 0$	(5)
$\beta(t) = \begin{cases} 1, & \text{if } t < 1, \\ 0, & \text{if } t > 1 \end{cases}$	$2 \operatorname{sinc}(\omega) = 2 \frac{\sin(\omega)}{\omega}$	Boxcar in time.	(6)
$\frac{1}{\pi} \operatorname{sinc}(t)$	$\beta(\omega)$	Boxcar in frequency.	(7)
$f'(t)$	$i\omega \widehat{f}(\omega)$	Derivative in time.	(8)
$f''(t)$	$(i\omega)^2 \widehat{f}(\omega)$	Higher derivatives similar.	(9)
$tf(t)$	$i \frac{d}{d\omega} \widehat{f}(\omega)$	Derivative in frequency.	(10)
$t^2 f(t)$	$i^2 \frac{d^2}{d\omega^2} \widehat{f}(\omega)$	Higher derivatives similar.	(11)
$e^{i\omega_0 t} f(t)$	$\widehat{f}(\omega - \omega_0)$	Modulation property.	(12)
$f\left(\frac{t - t_0}{k}\right)$	$ke^{-i\omega t_0} \widehat{f}(k\omega)$	Time shift and squeeze.	(13)
$(f * g)(t)$	$\widehat{f}(\omega)\widehat{g}(\omega)$	Convolution in time.	(14)
$u(t) = \begin{cases} 0, & \text{if } t < 0 \\ 1, & \text{if } t > 0 \end{cases}$	$\frac{1}{i\omega} + \pi\delta(\omega)$	Heaviside step function.	(15)
$\delta(t - t_0)f(t)$	$e^{-i\omega t_0} f(t_0)$	Assumes f continuous at t_0 .	(16)
$e^{i\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	Useful for $\sin(\omega_0 t)$, $\cos(\omega_0 t)$.	(17)

Convolution:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(t - u)g(u) du = \int_{-\infty}^{\infty} f(u)g(t - u) du.$$

Parseval:

$$\int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\widehat{f}(\omega)|^2 d\omega.$$

$j \frac{1}{\pi t}$	$\text{sgn}(\omega)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
$\sum_{n=-\infty}^{\infty} F_n e^{jn\omega_0 t}$	$2\pi \sum_{n=-\infty}^{\infty} F_n \delta(\omega - n\omega_0)$
$\text{rect}\left(\frac{t}{\tau}\right)$	$\tau \text{Sa}\left(\frac{\omega\tau}{2}\right)$
$\frac{B}{2\pi} \text{Sa}\left(\frac{Bt}{2}\right)$	$\text{rect}\left(\frac{\omega}{B}\right)$
$\text{tri}(t)$	$\text{Sa}^2\left(\frac{\omega}{2}\right)$
$A \cos\left(\frac{\pi t}{2\tau}\right) \text{rect}\left(\frac{t}{2\tau}\right)$	$\frac{A\pi}{\tau} \frac{\cos(\omega\tau)}{(\pi/2\tau)^2 - \omega^2}$
$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$u(t) \cos(\omega_0 t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
$u(t) \sin(\omega_0 t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega^2}{\omega_0^2 - \omega^2}$
$u(t)e^{-\alpha t} \cos(\omega_0 t)$	$\frac{(\alpha + j\omega)}{\omega_0^2 + (\alpha + j\omega)^2}$

$u(t)e^{-\alpha t} \sin(\omega_0 t)$	$\frac{\omega_0}{\omega_0^2 + (\alpha + j\omega)^2}$
$e^{-\alpha t }$	$\frac{2\alpha}{\alpha^2 + \omega^2}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi} e^{-\sigma^2\omega^2/2}$
$u(t)e^{-\alpha t}$	$\frac{1}{\alpha + j\omega}$
$u(t)te^{-\alpha t}$	$\frac{1}{(\alpha + j\omega)^2}$

➤ **Trigonometric Fourier Series**

$$f(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos(\omega_0 nt) + b_n \sin(\omega_0 nt))$$

where

$$a_0 = \frac{1}{T} \int_0^T f(t) dt \quad , \quad a_n = \frac{2}{T} \int_0^T f(t) \cos(\omega_0 nt) dt \quad , \text{ and}$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(\omega_0 nt) dt$$

➤ **Complex Exponential Fourier Series**

$$f(t) = \sum_{n=-\infty}^{\infty} F_n e^{j\omega_0 nt} \quad , \text{ where} \quad F_n = \frac{1}{T} \int_0^T f(t) e^{-j\omega_0 nt} dt$$

Some Useful Mathematical Relationships

$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$
$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$
$\cos(x \pm y) = \cos(x) \cos(y) \mp \sin(x) \sin(y)$
$\sin(x \pm y) = \sin(x) \cos(y) \pm \cos(x) \sin(y)$
$\cos(2x) = \cos^2(x) - \sin^2(x)$
$\sin(2x) = 2 \sin(x) \cos(x)$
$2 \cos^2(x) = 1 + \cos(2x)$
$2 \sin^2(x) = 1 - \cos(2x)$
$\cos^2(x) + \sin^2(x) = 1$
$2 \cos(x) \cos(y) = \cos(x - y) + \cos(x + y)$
$2 \sin(x) \sin(y) = \cos(x - y) - \cos(x + y)$
$2 \sin(x) \cos(y) = \sin(x - y) + \sin(x + y)$

Useful Integrals

$\int \cos(x) dx$	$\sin(x)$
$\int \sin(x) dx$	$-\cos(x)$
$\int x \cos(x) dx$	$\cos(x) + x \sin(x)$
$\int x \sin(x) dx$	$\sin(x) - x \cos(x)$
$\int x^2 \cos(x) dx$	$2x \cos(x) + (x^2 - 2) \sin(x)$
$\int x^2 \sin(x) dx$	$2x \sin(x) - (x^2 - 2) \cos(x)$
$\int e^{\alpha x} dx$	$\frac{e^{\alpha x}}{\alpha}$
$\int x e^{\alpha x} dx$	$e^{\alpha x} \left[\frac{x}{\alpha} - \frac{1}{\alpha^2} \right]$
$\int x^2 e^{\alpha x} dx$	$e^{\alpha x} \left[\frac{x^2}{\alpha} - \frac{2x}{\alpha^2} + \frac{2}{\alpha^3} \right]$
$\int \frac{dx}{\alpha + \beta x}$	$\frac{1}{\beta} \ln \alpha + \beta x $
$\int \frac{dx}{\alpha^2 + \beta^2 x^2}$	$\frac{1}{\alpha\beta} \tan^{-1} \left(\frac{\beta x}{\alpha} \right)$

Engineering Tables/Fourier Transform Table 2

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Signal	Fourier transform unitary, angular frequency	Fourier transform unitary, ordinary frequency	Remarks
$g(t) \equiv$	$G(\omega) \equiv$	$G(f) \equiv$	
10 $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} G(\omega)e^{i\omega t} d\omega$ $\text{rect}(at)$	$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(t)e^{-i\omega t} dt$ $\frac{1}{\sqrt{2\pi a^2}} \cdot \text{sinc}\left(\frac{\omega}{2\pi a}\right)$	$\int_{-\infty}^{\infty} g(t)e^{-i2\pi ft} dt$ $\frac{1}{ a } \cdot \text{sinc}\left(\frac{f}{a}\right)$	The rectangular pulse and the normalized sinc function
11 $\text{sinc}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \text{rect}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{ a } \cdot \text{rect}\left(\frac{f}{a}\right)$	Dual of rule 10. The rectangular function is an idealized low-pass filter, and the sinc function is the non-causal impulse response of such a filter.
12 $\text{sinc}^2(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \text{tri}\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{ a } \cdot \text{tri}\left(\frac{f}{a}\right)$	tri is the triangular function
13 $\text{tri}(at)$	$\frac{1}{\sqrt{2\pi a^2}} \cdot \text{sinc}^2\left(\frac{\omega}{2\pi a}\right)$	$\frac{1}{ a } \cdot \text{sinc}^2\left(\frac{f}{a}\right)$	Dual of rule 12.
14 $e^{-\alpha t^2}$	$\frac{1}{\sqrt{2\alpha}} \cdot e^{-\frac{\omega^2}{4\alpha}}$	$\sqrt{\frac{\pi}{\alpha}} \cdot e^{-\frac{(\pi f)^2}{\alpha}}$	Shows that the Gaussian function $\exp(-a^2 t^2)$ is its own Fourier transform. For this to be integrable we must have $\text{Re}(a) > 0$.

$$e^{iat^2} = e^{-at^2} \Big|_{\alpha=-ia}$$

common in optics

$$\begin{aligned} \cos(at^2) & \frac{1}{\sqrt{2a}} \cdot e^{-i\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right)} & \sqrt{\frac{\pi}{a}} \cdot e^{-i\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right)} \\ \sin(at^2) & \frac{1}{\sqrt{2a}} \cos\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right) & \sqrt{\frac{\pi}{a}} \cos\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right) \\ & \frac{-1}{\sqrt{2a}} \sin\left(\frac{\omega^2}{4a} - \frac{\pi}{4}\right) & -\sqrt{\frac{\pi}{a}} \sin\left(\frac{\pi^2 f^2}{a} - \frac{\pi}{4}\right) \end{aligned}$$

$$e^{-a|t|} \quad \sqrt{\frac{2}{\pi}} \cdot \frac{a}{a^2 + \omega^2} \quad \frac{2a}{a^2 + 4\pi^2 f^2} \quad a > 0$$

$$\frac{1}{\sqrt{|t|}} \quad \frac{1}{\sqrt{|\omega|}} \quad \frac{1}{\sqrt{|f|}}$$

the transform is the function itself

$$J_0(t) \quad \sqrt{\frac{2}{\pi}} \cdot \frac{\text{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1 - \omega^2}} \quad \frac{2 \cdot \text{rect}(\pi f)}{\sqrt{1 - 4\pi^2 f^2}}$$

$J_0(t)$ is the Bessel function of first kind of order 0, rect is the rectangular function

$$J_n(t) \quad \sqrt{\frac{2}{\pi}} \frac{(-i)^n T_n(\omega) \text{rect}\left(\frac{\omega}{2}\right)}{\sqrt{1 - \omega^2}} \quad \frac{2(-i)^n T_n(2\pi f) \text{rect}(\pi f)}{\sqrt{1 - 4\pi^2 f^2}}$$

it's the generalization of the previous transform; $T_n(t)$ is the Chebyshev polynomial of the first kind.

$$\frac{J_n(t)}{t} \quad \sqrt{\frac{2}{\pi n}} \frac{(-i)^n \cdot U_{n-1}(\omega)}{\sqrt{1 - \omega^2} \text{rect}\left(\frac{\omega}{2}\right)} \quad \frac{2i}{n} \frac{(-i)^n \cdot U_{n-1}(2\pi f)}{\sqrt{1 - 4\pi^2 f^2} \text{rect}(\pi f)}$$

$U_n(t)$ is the Chebyshev polynomial of the second kind

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