The normal distribution

INTRODUCTION TO STATISTICS IN PYTHON

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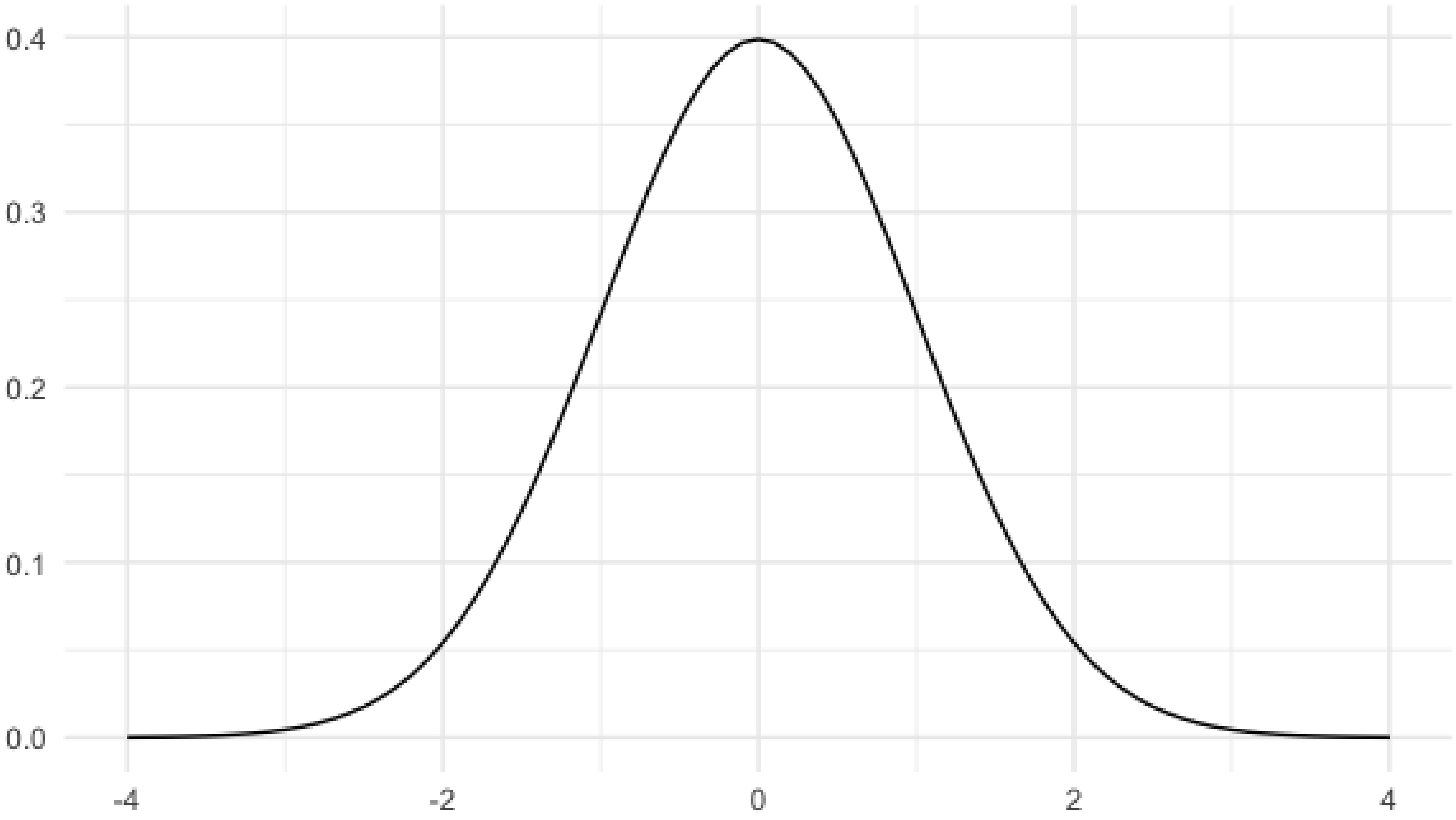
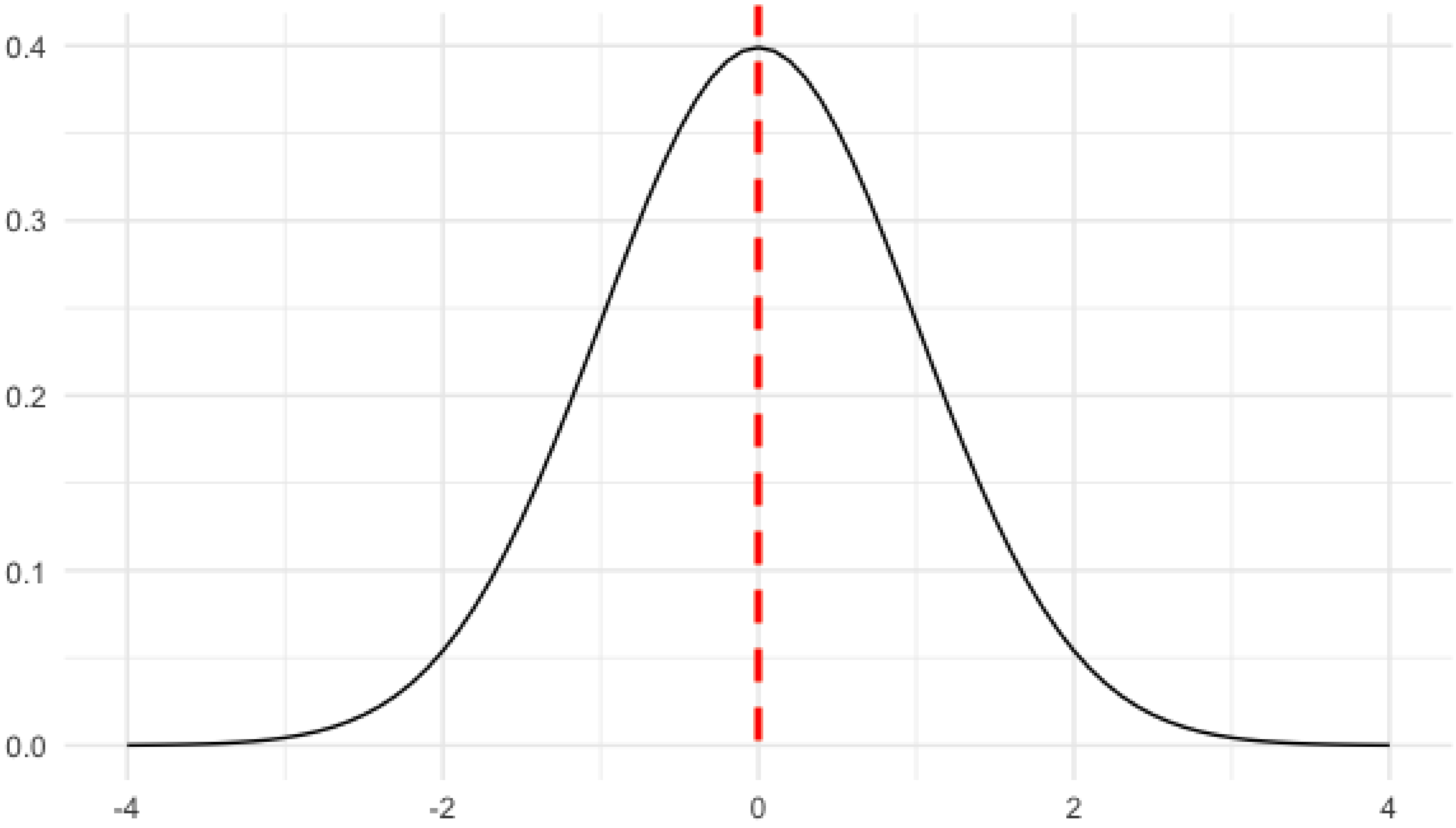
datacamp

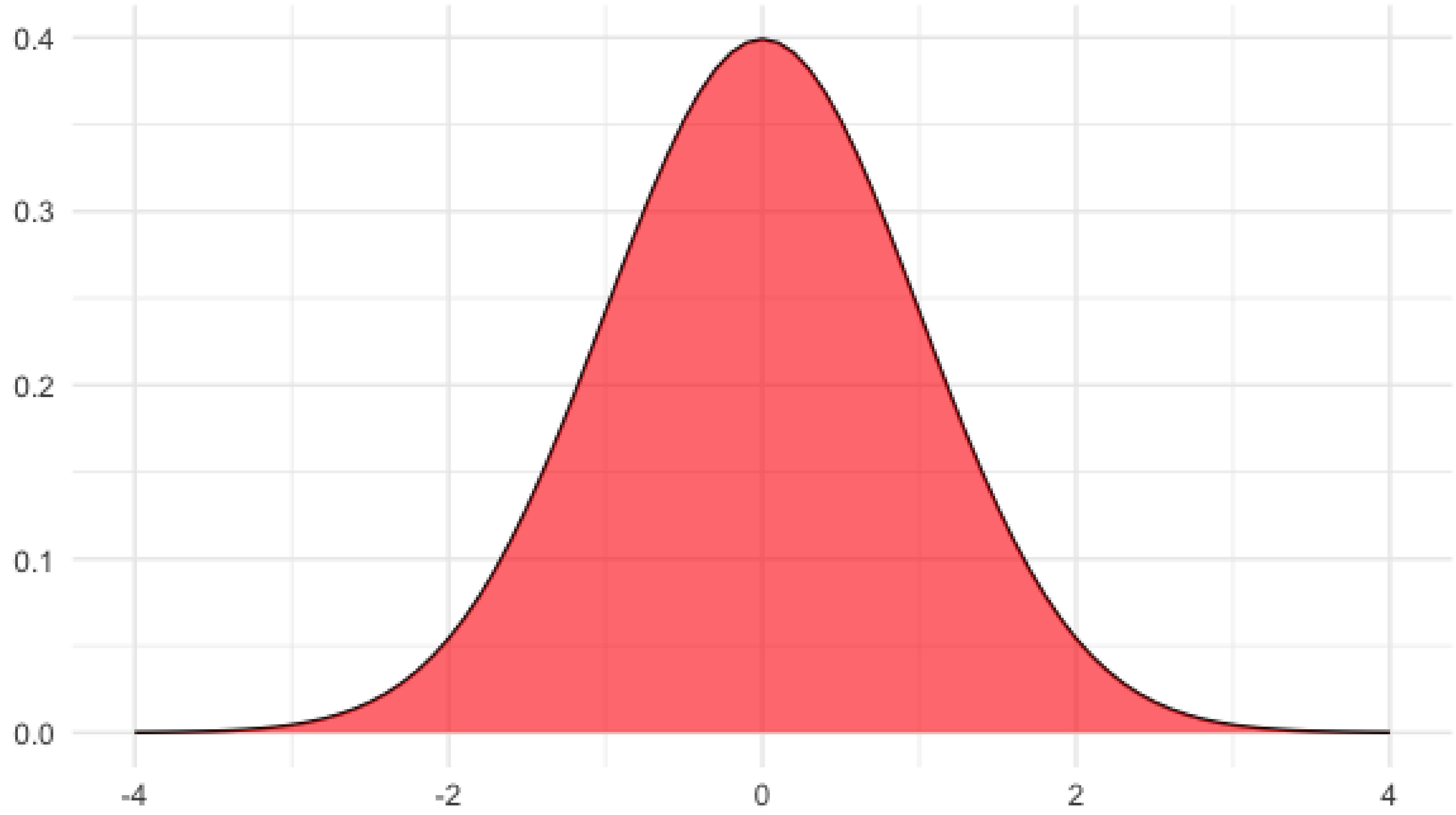
What is the normal distribution?



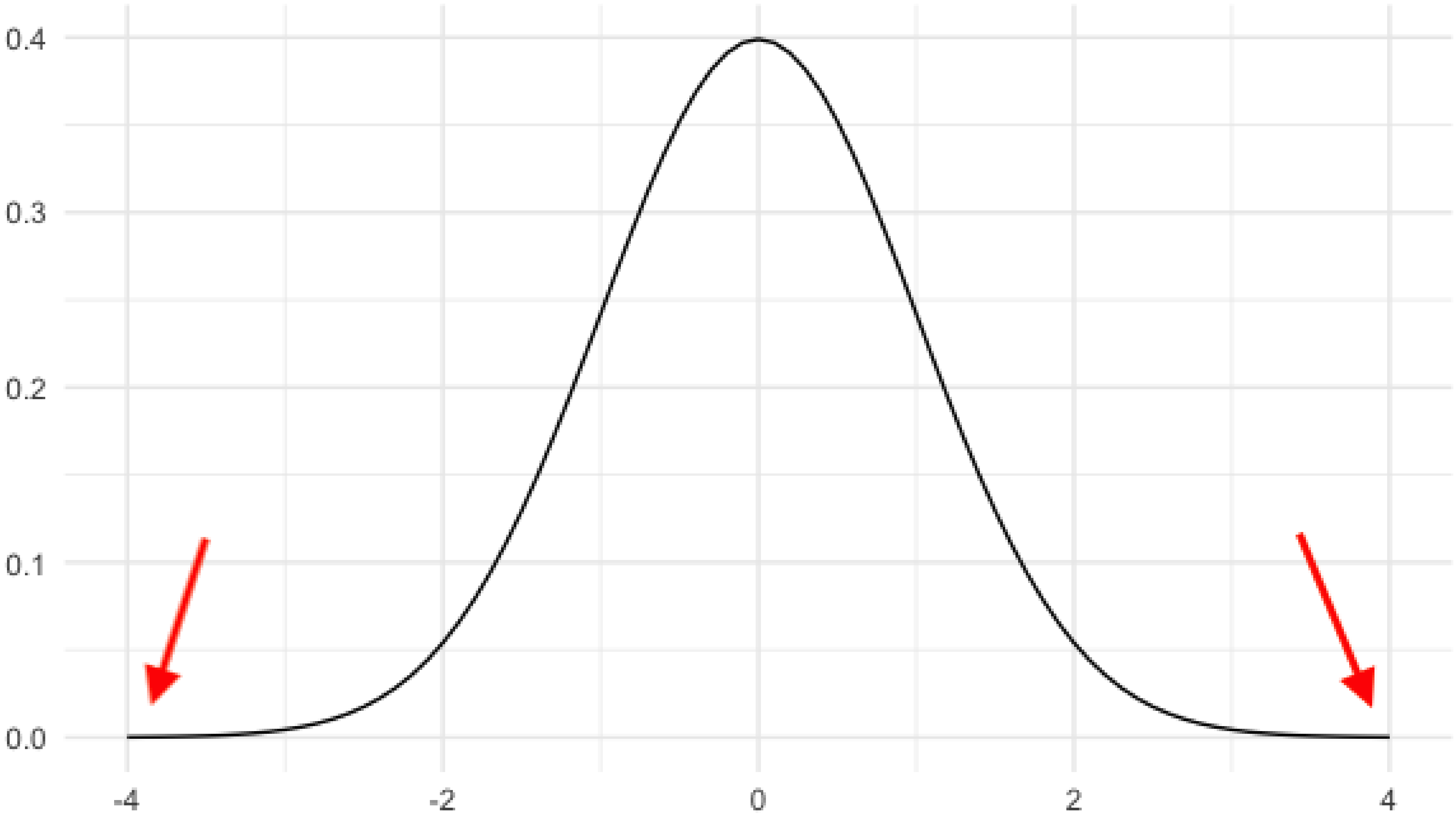
Symmetrical



Area = 1



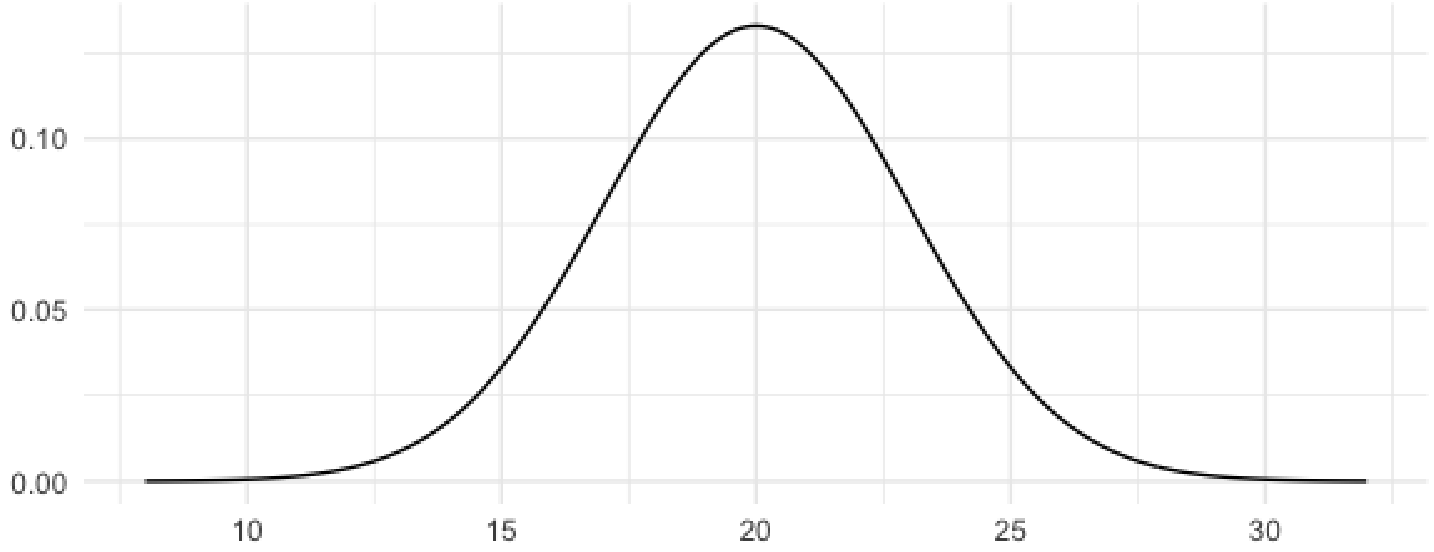
Curve never hits 0



Described by mean and standard deviation

Mean: 20

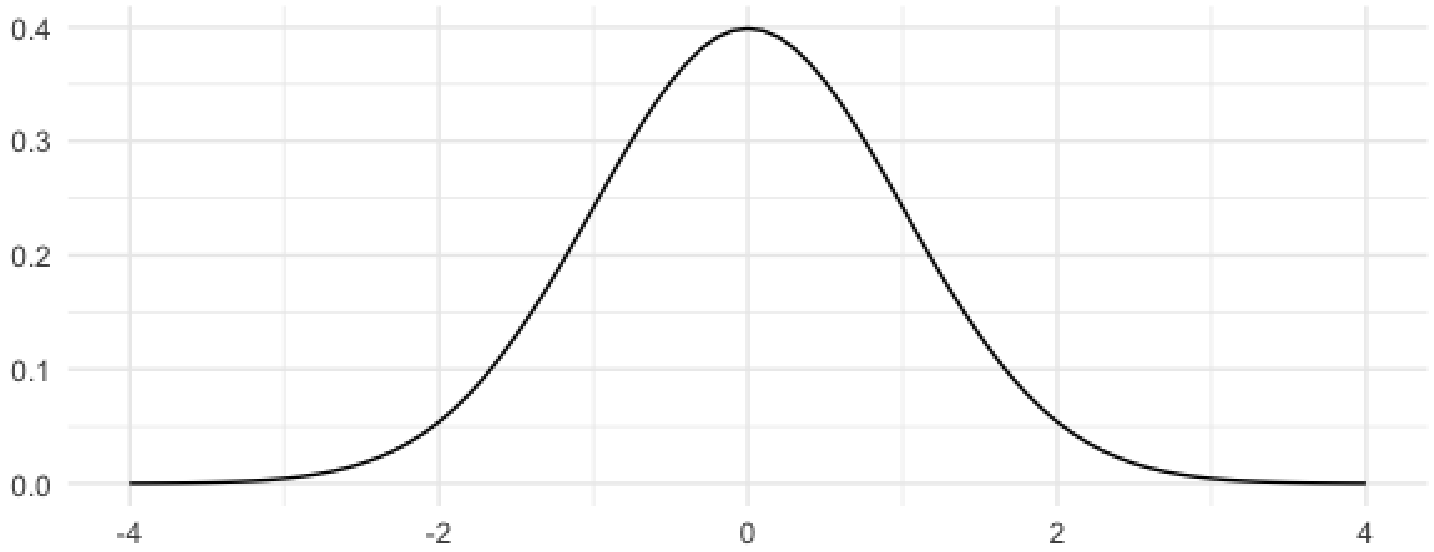
Standard deviation: 3



Standard normal distribution

Mean: 0

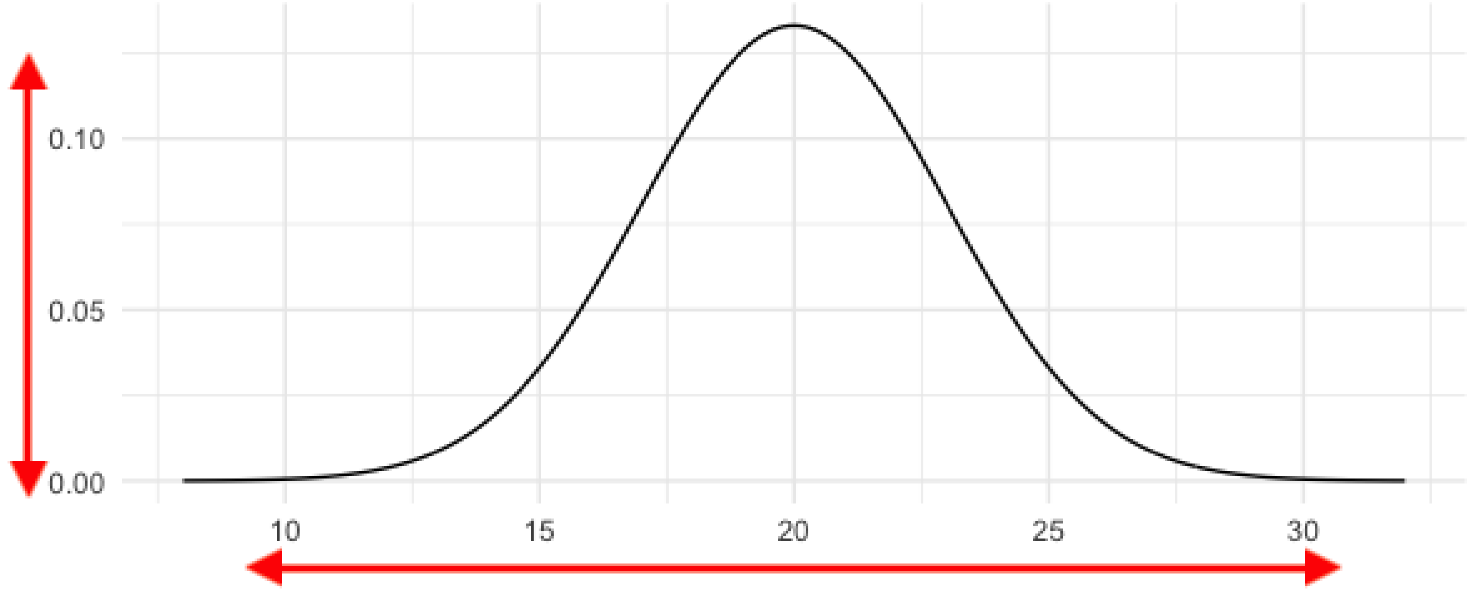
Standard deviation: 1



Described by mean and standard deviation

Mean: 20

Standard deviation: 3



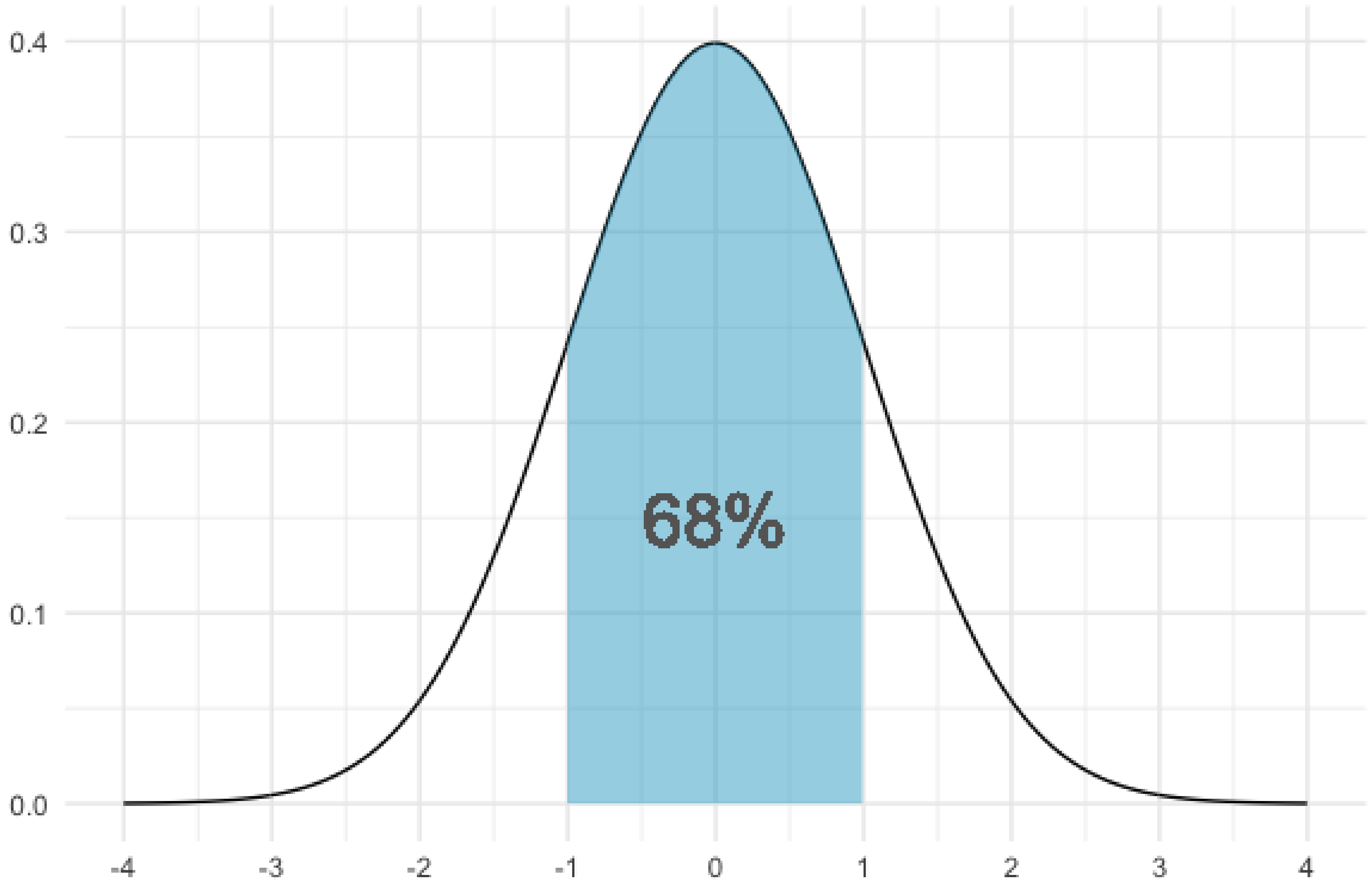
Standard normal distribution

Mean: 0

Standard deviation: 1

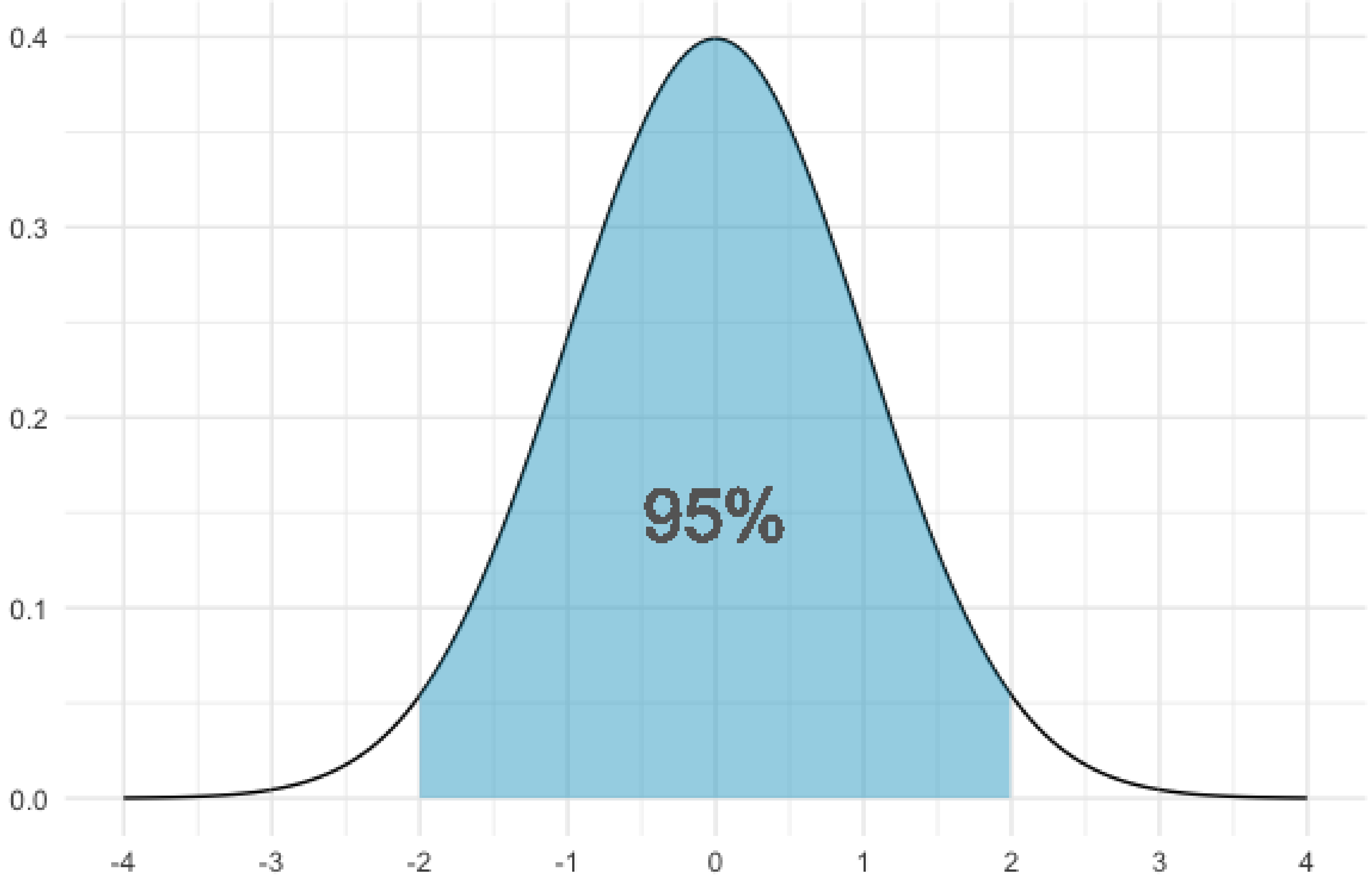
Areas under the normal distribution

68% falls within 1 standard deviation



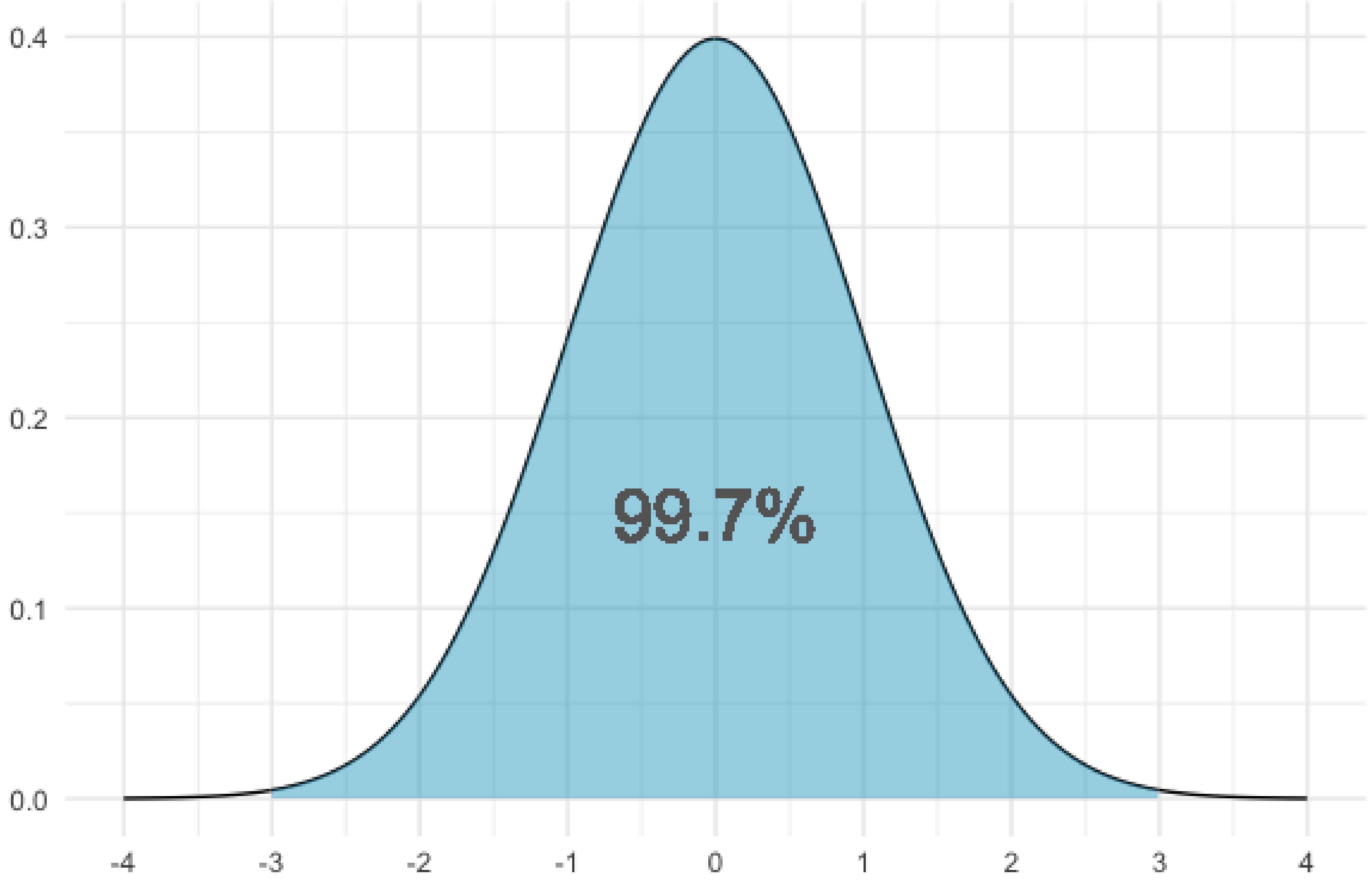
Areas under the normal distribution

95% falls within 2 standard deviations



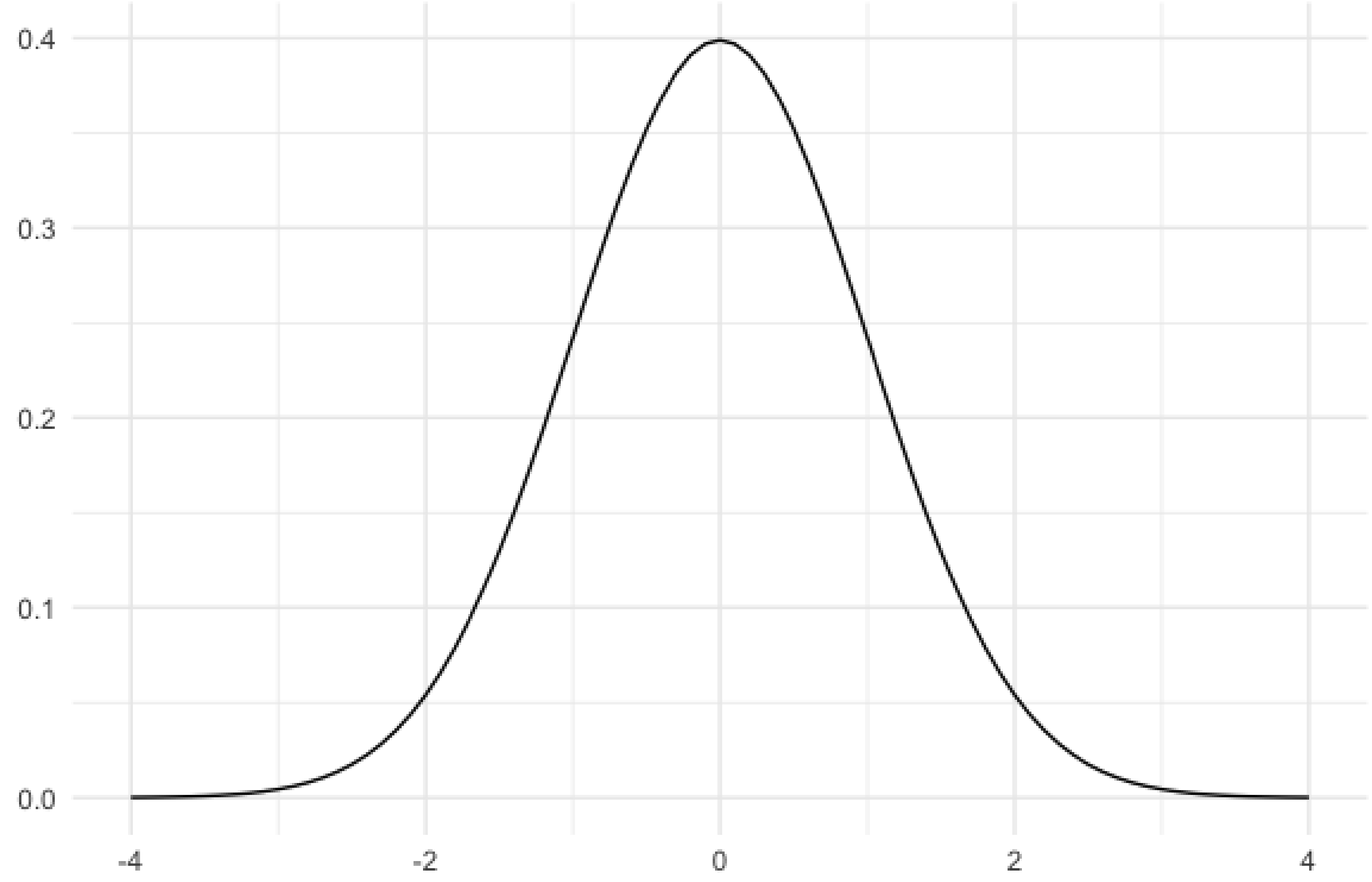
Areas under the normal distribution

99.7% falls within 3 standard deviations

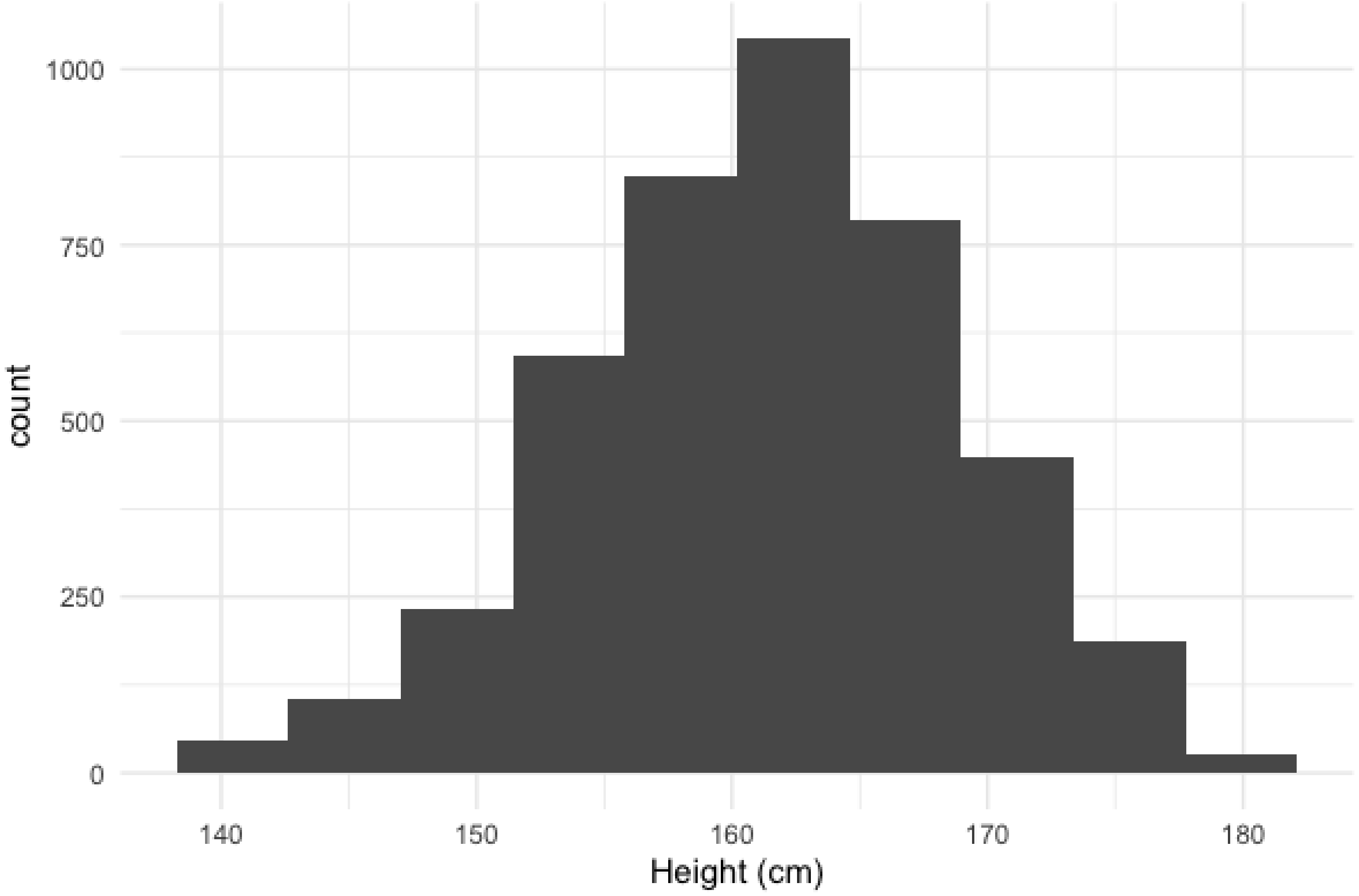


Lots of histograms look normal

Normal distribution

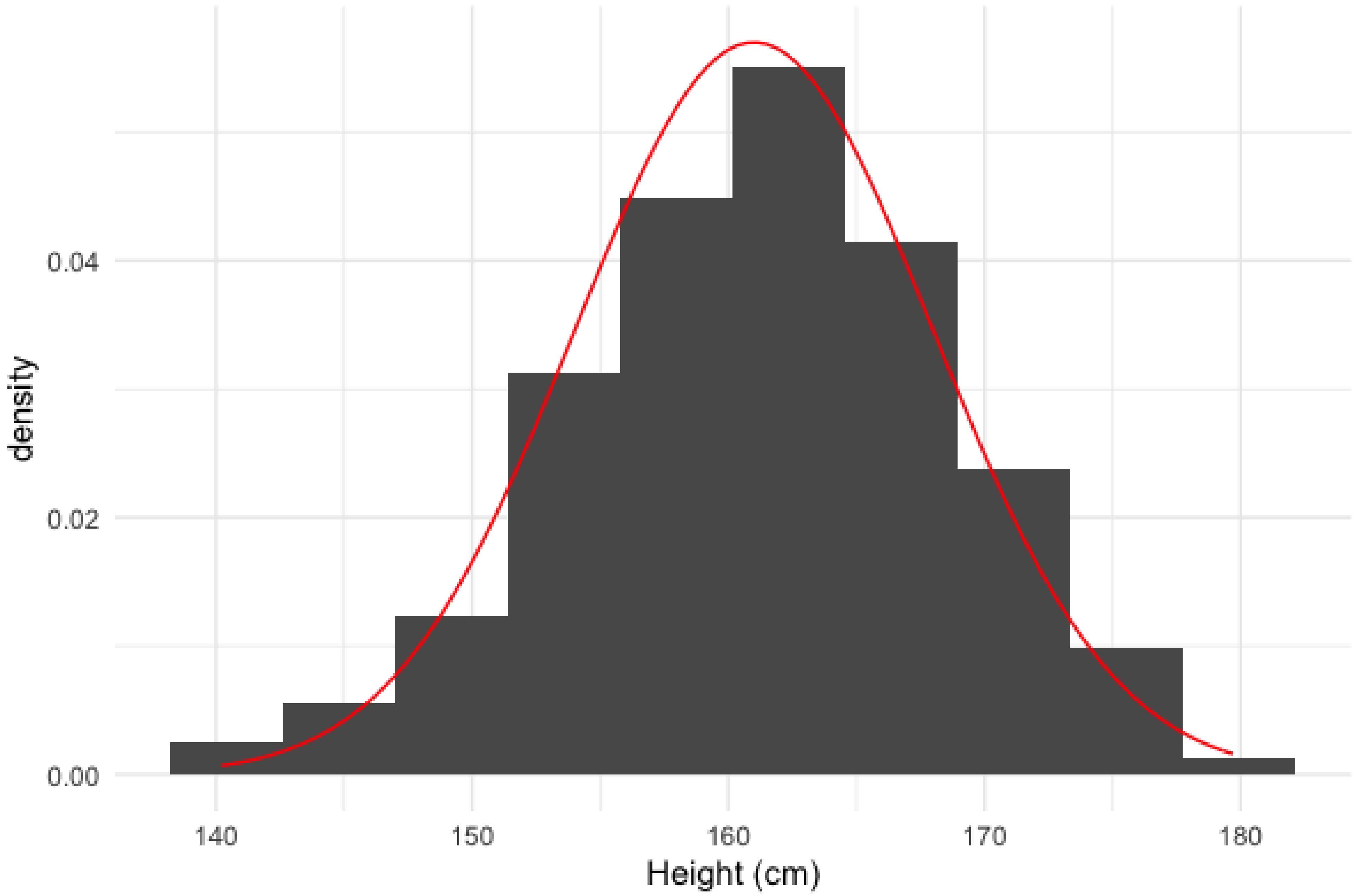


Women's heights from NHANES

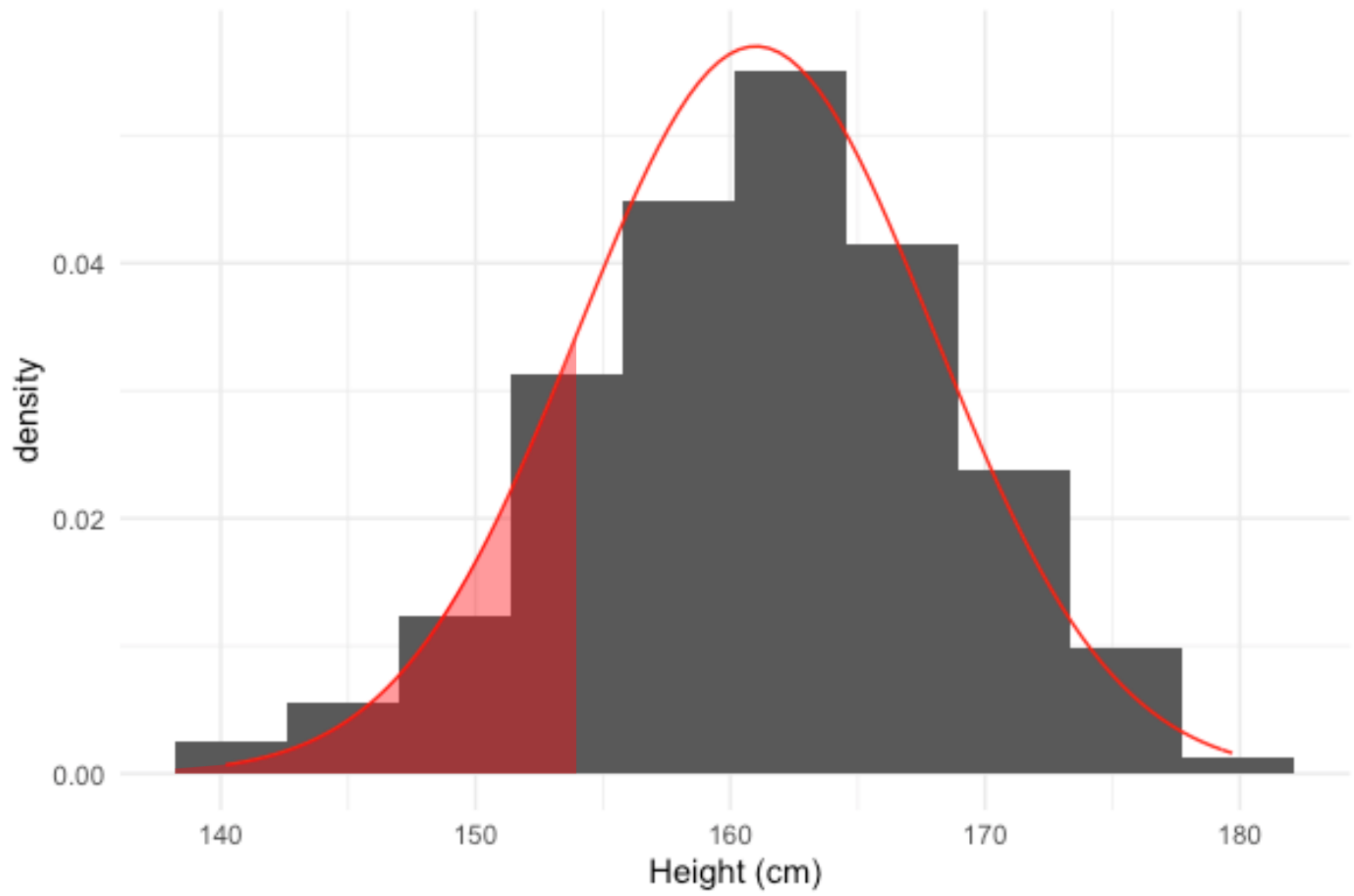


Mean: 161 cm *Standard deviation: 7 cm*

Approximating data with the normal distribution



What percent of women are shorter than 154 cm?

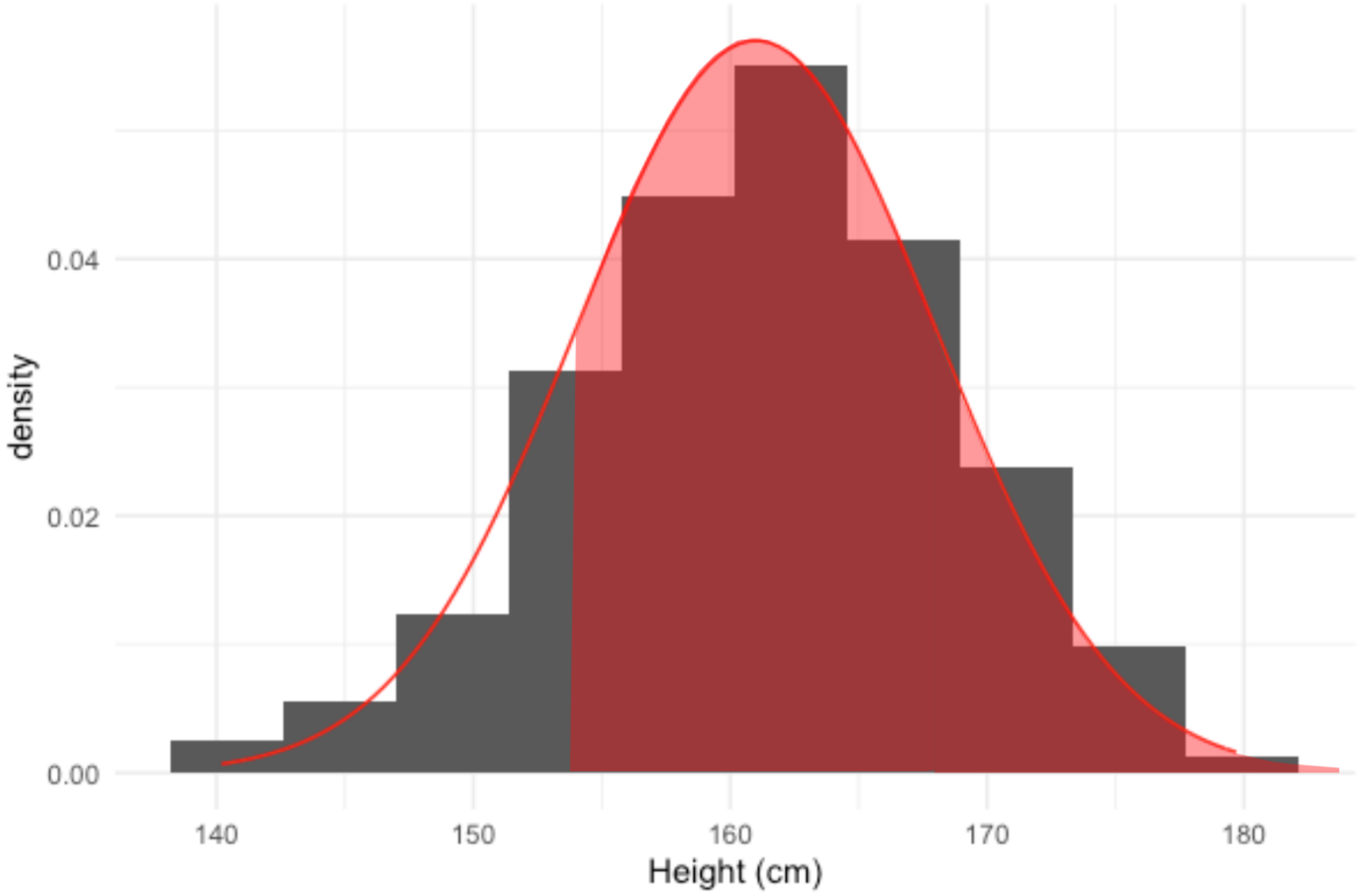



```python
from scipy.stats import norm
norm.cdf(154, 161, 7)
```

```
0.158655
```

16% of women in the survey are shorter than 154 cm

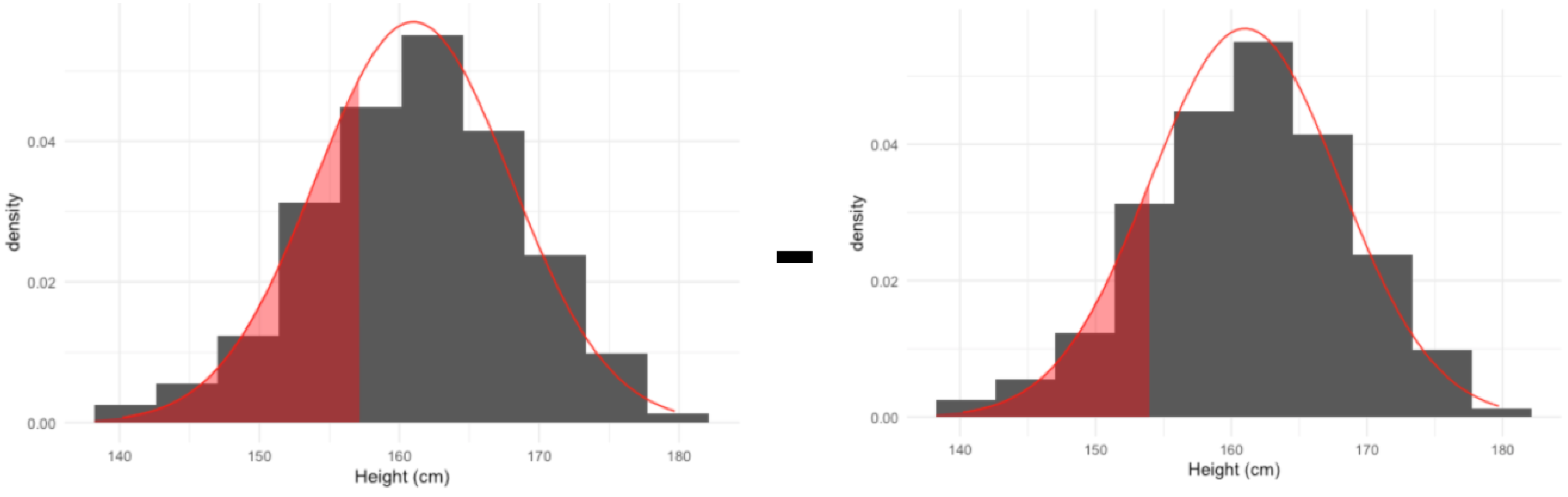
What percent of women are taller than 154 cm?




```python
from scipy.stats import norm
1 - norm.cdf(154, 161, 7)
```

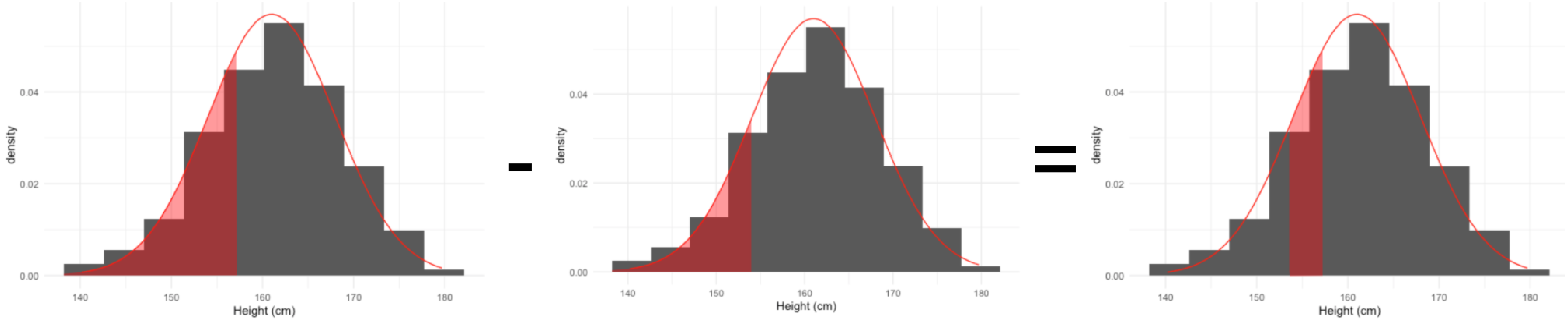
```
0.841345
```

What percent of women are 154-157 cm?




```
norm.cdf(157, 161, 7) - norm.cdf(154, 161, 7)
```

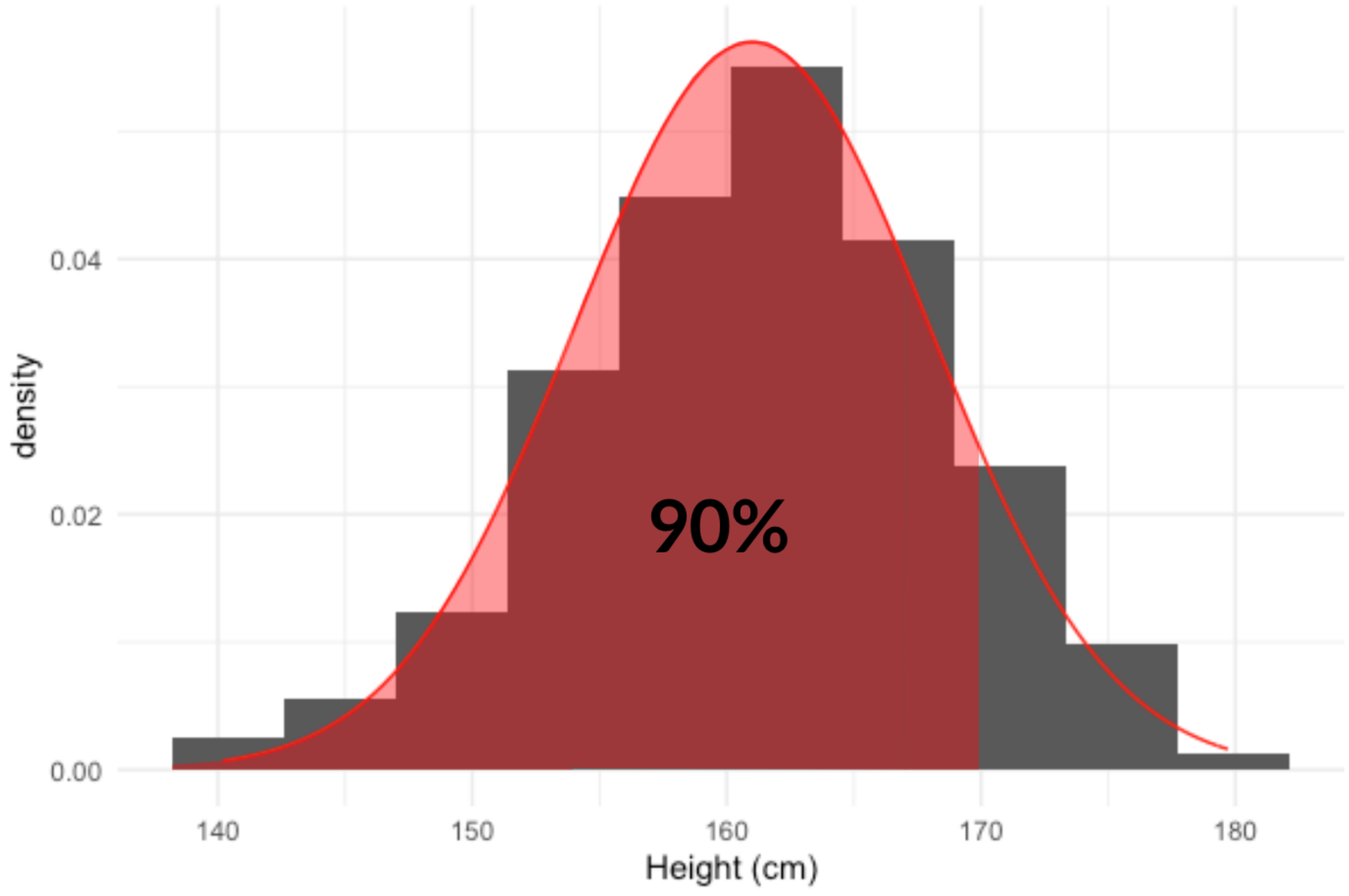
What percent of women are 154-157 cm?




```
norm.cdf(157, 161, 7) - norm.cdf(154, 161, 7)
```

```
0.1252
```

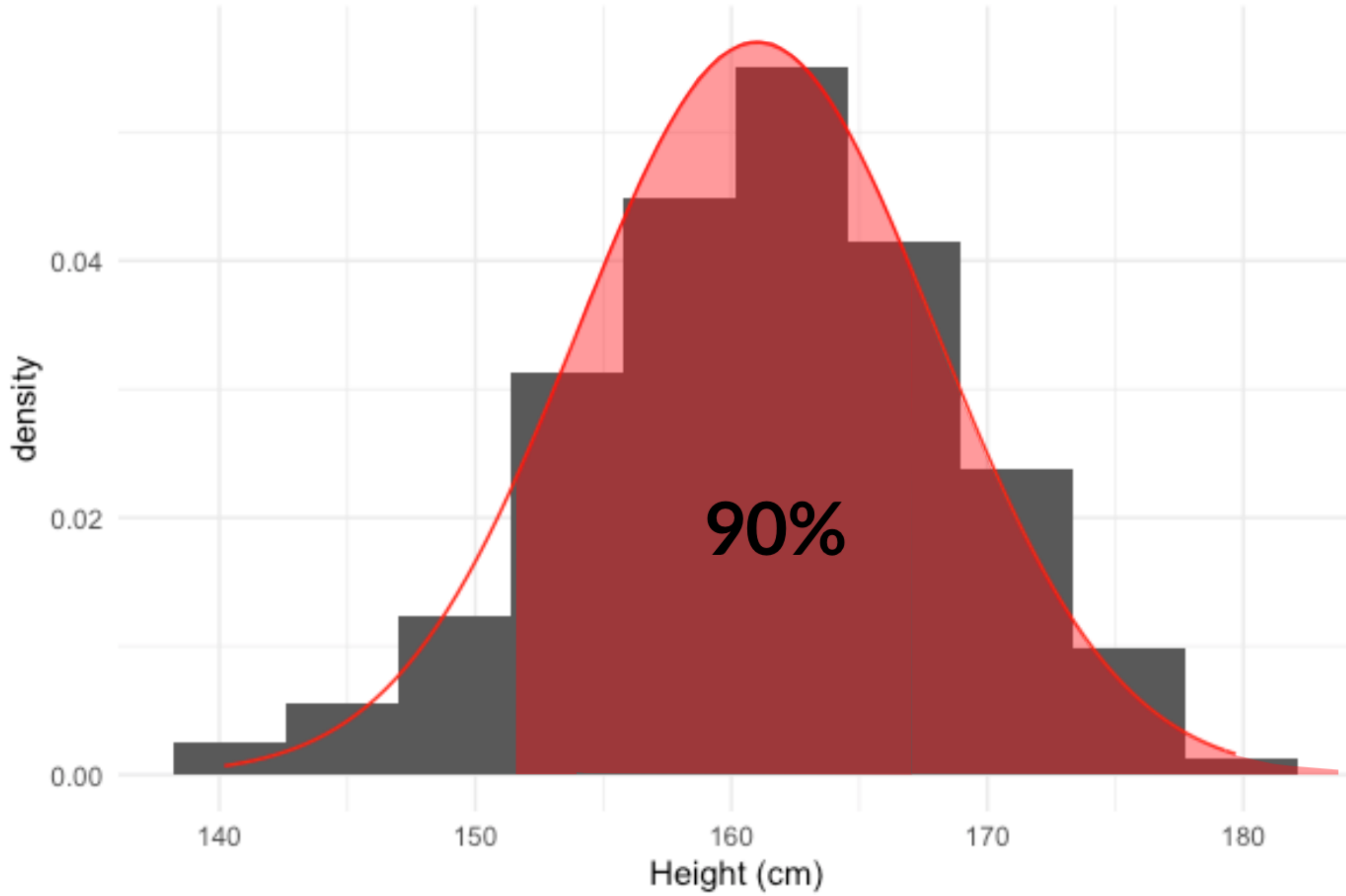
What height are 90% of women shorter than?




```
norm.ppf(0.9, 161, 7)
```

```
169.97086
```

What height are 90% of women taller than?




```
norm.ppf((1-0.9), 161, 7)
```

```
152.029
```

Generating random numbers

```
# Generate 10 random heights
norm.rvs(161, 7, size=10)
```

```
array([155.5758223 , 155.13133235, 160.06377097, 168.33345778,
       165.92273375, 163.32677057, 165.13280753, 146.36133538,
       149.07845021, 160.5790856 ])
```

Let's practice!

The central limit theorem

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Rolling the dice 5 times

```
die = pd.Series([1, 2, 3, 4, 5, 6])
# Roll 5 times
samp_5 = die.sample(5, replace=True)
print(samp_5)
```

```
array([3, 1, 4, 1, 1])
```

```
np.mean(samp_5)
```

```
2.0
```

Rolling the dice 5 times

```
# Roll 5 times and take mean
samp_5 = die.sample(5, replace=True)
np.mean(samp_5)
```

```
4.4
```

```
samp_5 = die.sample(5, replace=True)
np.mean(samp_5)
```

```
3.8
```

Rolling the dice 5 times 10 times

Repeat 10 times:

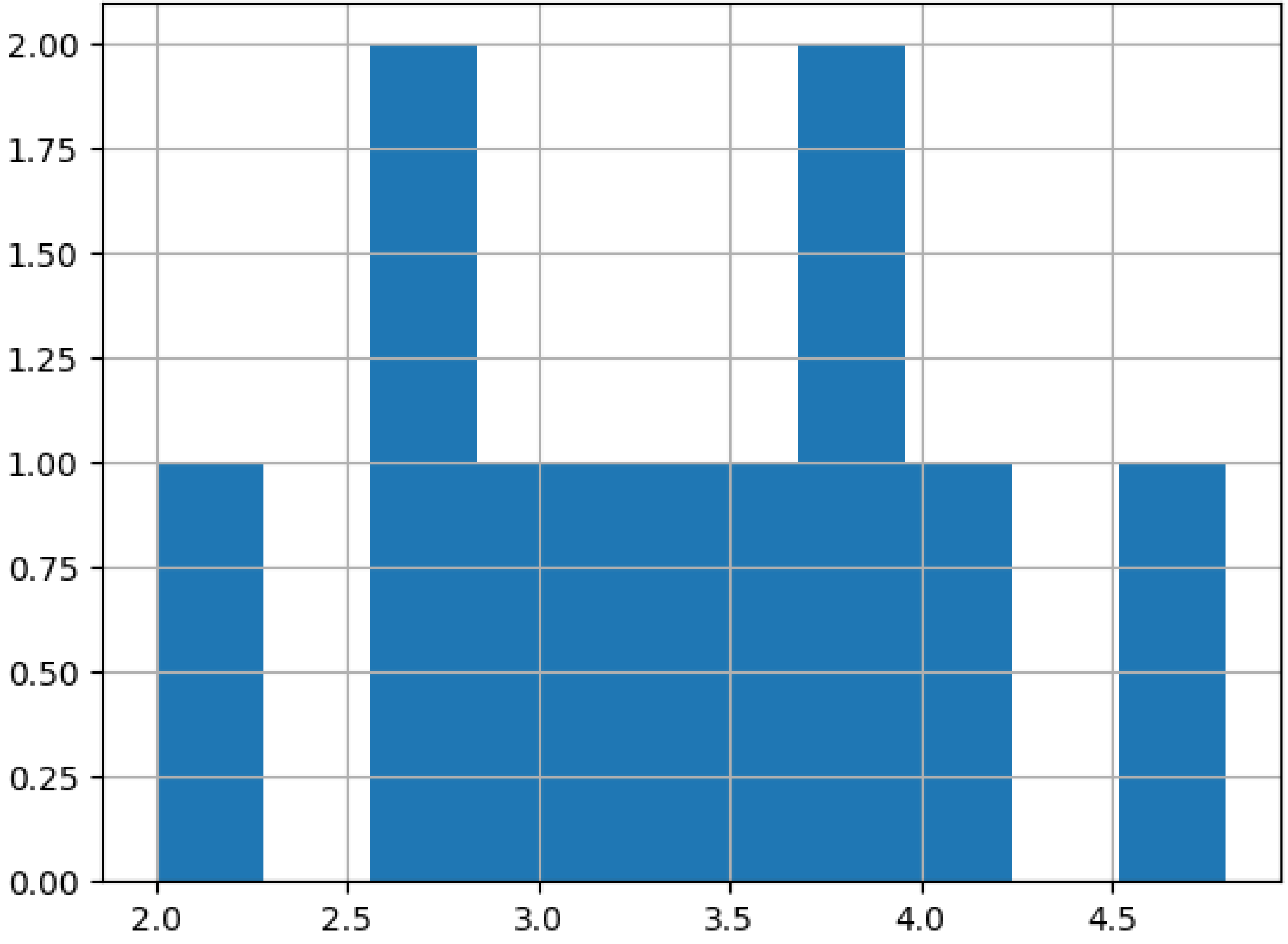
- Roll 5 times
- Take the mean

```
sample_means = []
for i in range(10):
  samp_5 = die.sample(5, replace=True)
  sample_means.append(np.mean(samp_5))
print(sample_means)
```

```
[3.8, 4.0, 3.8, 3.6, 3.2, 4.8, 2.6,
3.0, 2.6, 2.0]
```

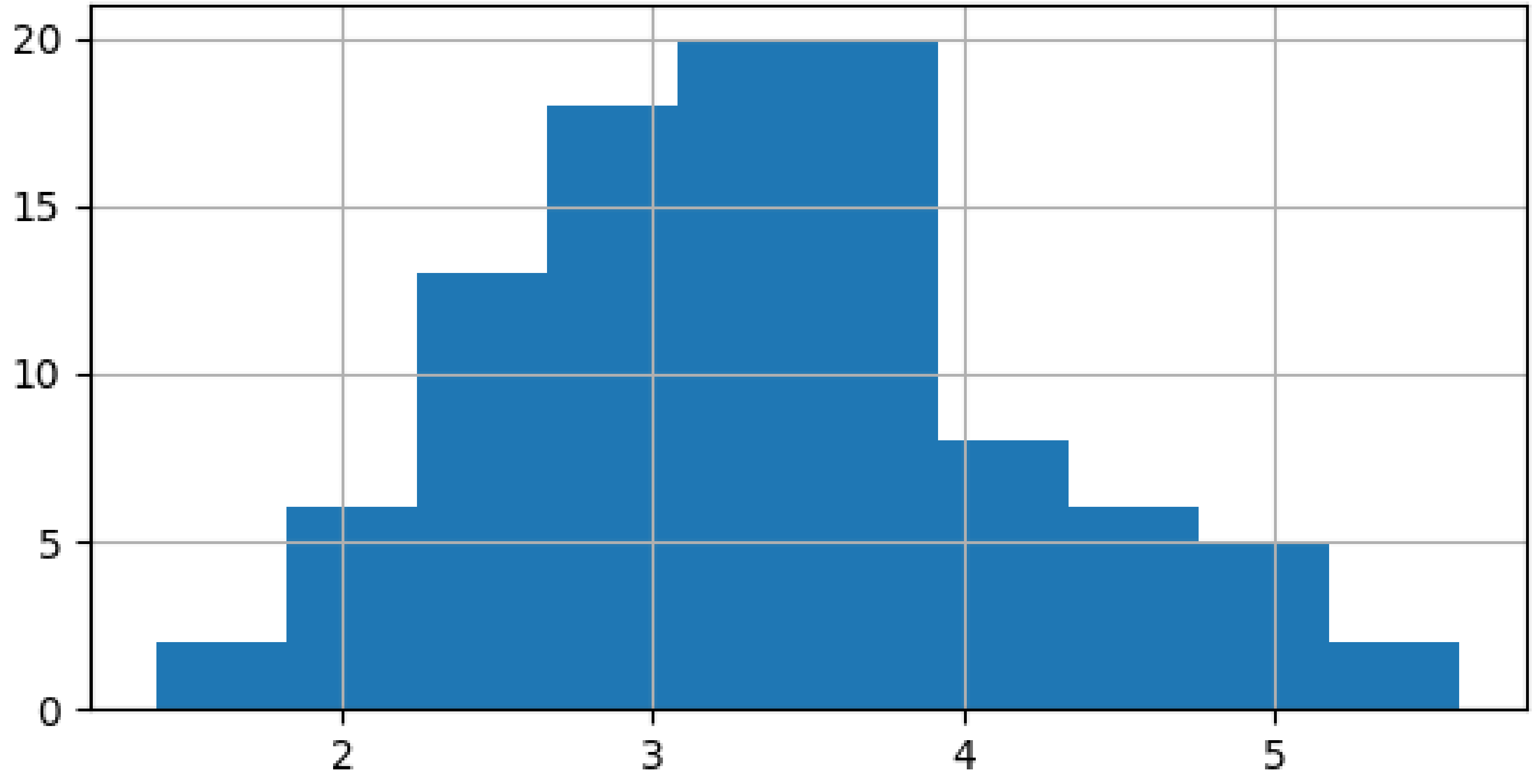
Sampling distributions

Sampling distribution of the sample mean



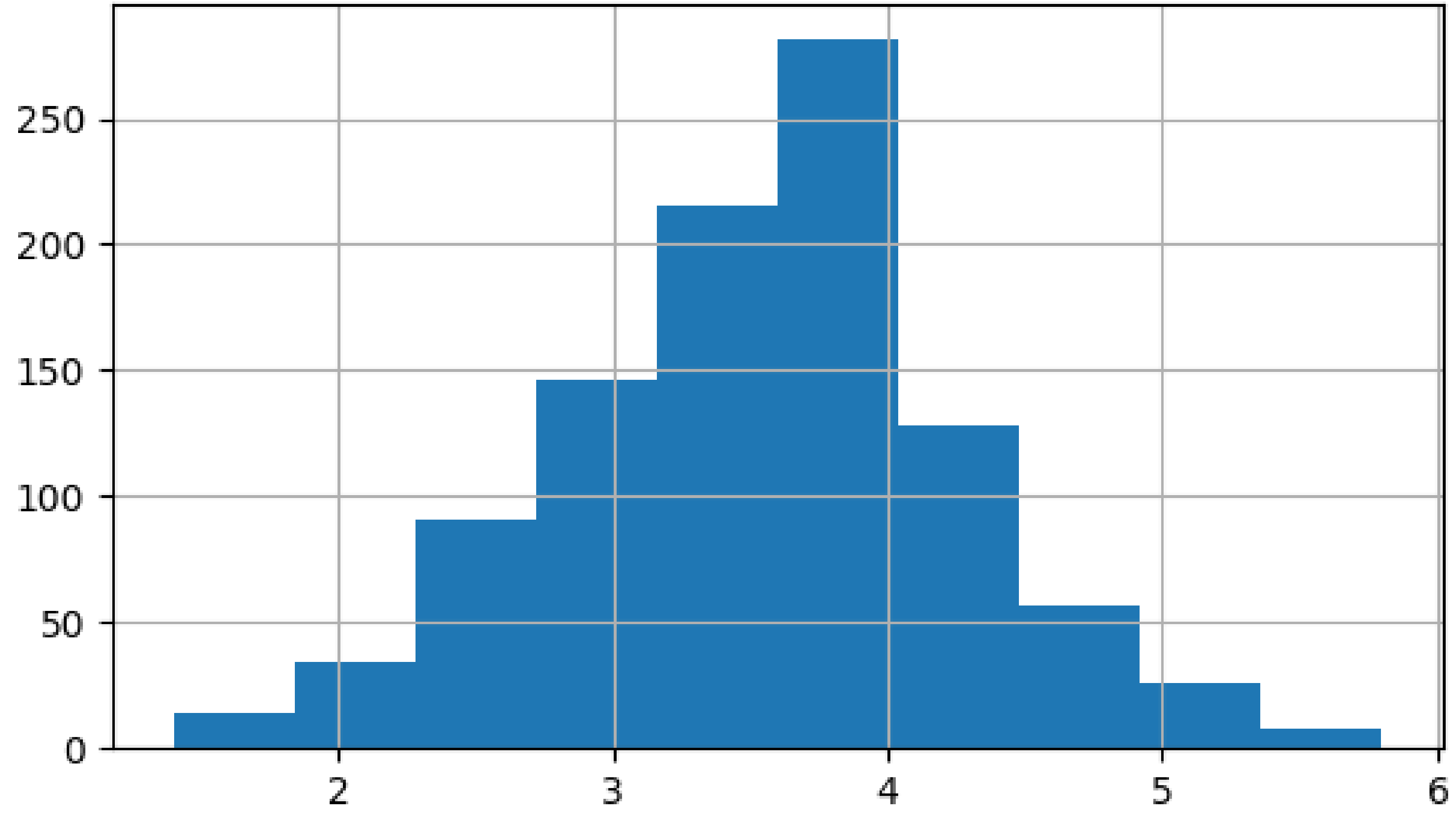
100 sample means

```
sample_means = []
for i in range(100):
    sample_means.append(np.mean(die.sample(5, replace=True)))
```



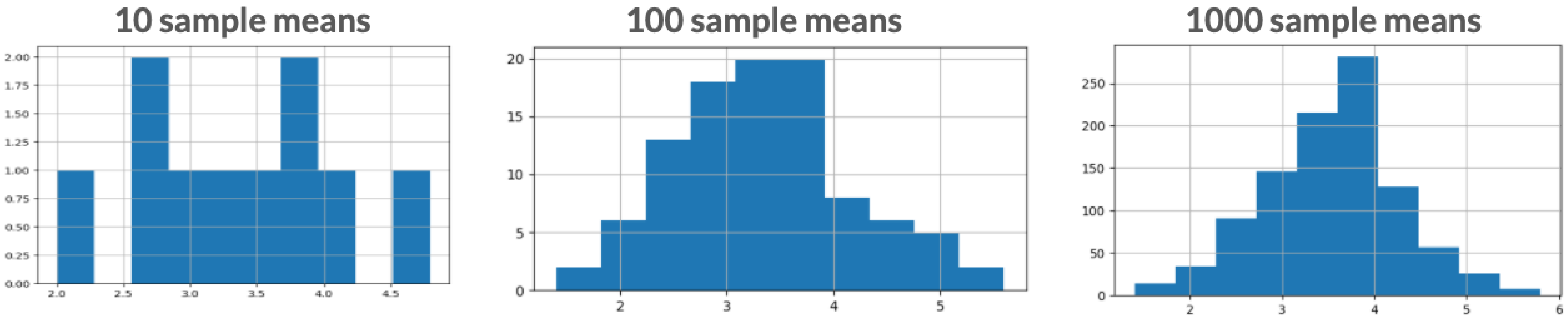
1000 sample means

```
sample_means = []
for i in range(1000):
    sample_means.append(np.mean(die.sample(5, replace=True)))
```



Central limit theorem

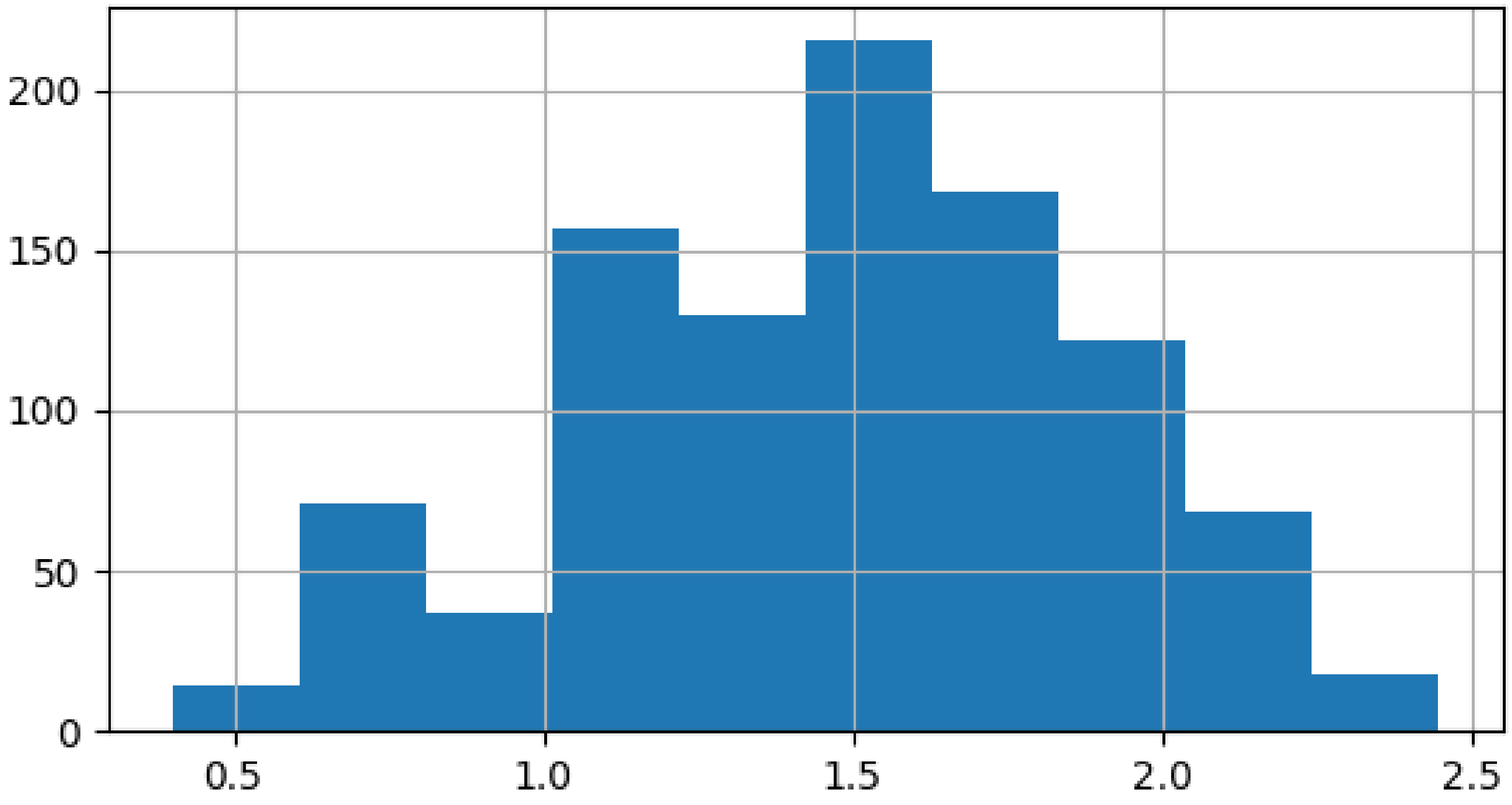
The sampling distribution of a statistic becomes closer to the normal distribution as the number of trials increases.



** Samples should be random and independent*

Standard deviation and the CLT

```
sample_sds = []
for i in range(1000):
  sample_sds.append(np.std(die.sample(5, replace=True)))
```



Proportions and the CLT

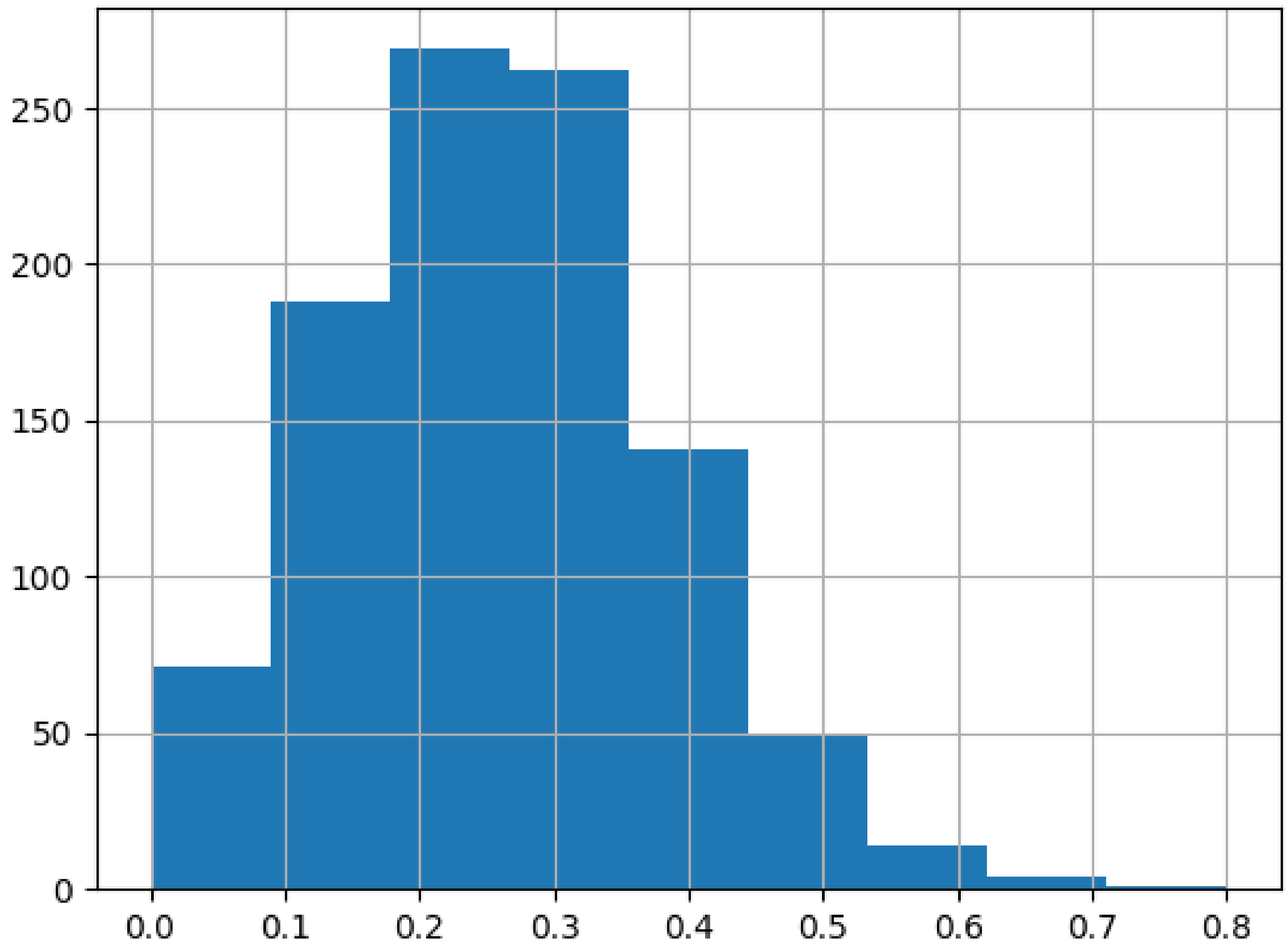
```
sales_team = pd.Series(["Amir", "Brian", "Claire", "Damian"])
sales_team.sample(10, replace=True)
```

```
array(['Claire', 'Damian', 'Brian', 'Damian', 'Damian', 'Amir', 'Amir', 'Amir',
       'Amir', 'Damian'], dtype=object)
```

```
sales_team.sample(10, replace=True)
```

```
array(['Brian', 'Amir', 'Brian', 'Claire', 'Brian', 'Damian', 'Claire', 'Brian',
       'Claire', 'Claire'], dtype=object)
```

Sampling distribution of proportion



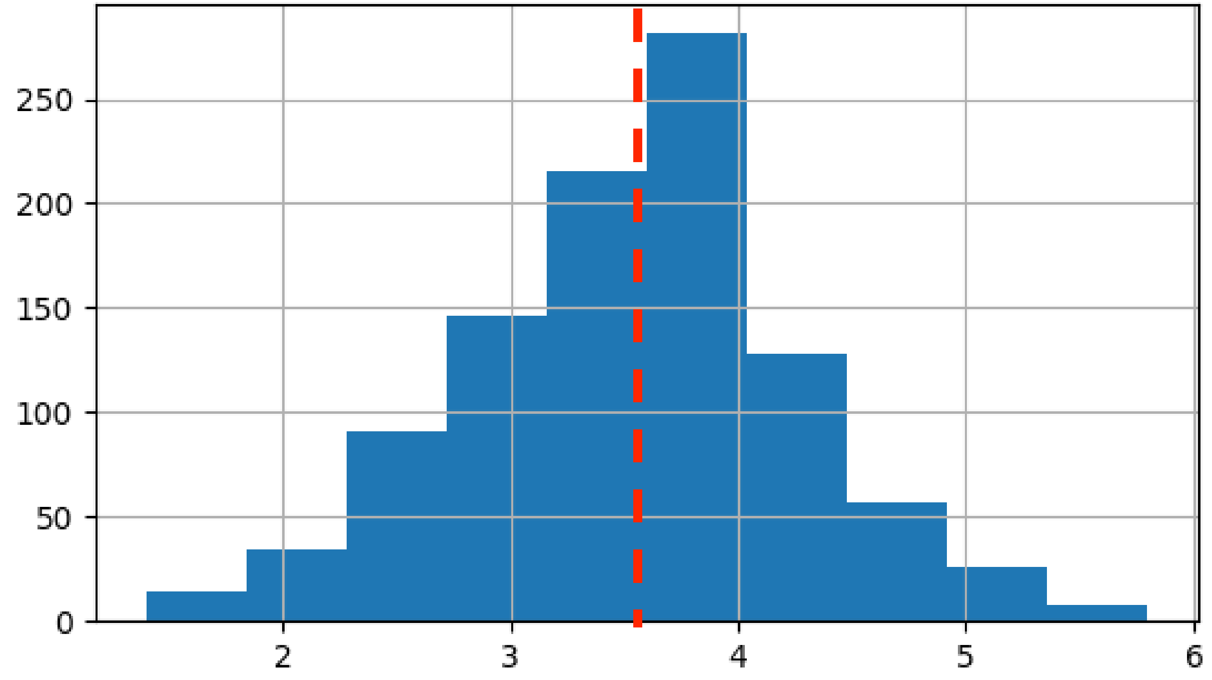
Mean of sampling distribution

```
# Estimate expected value of die
np.mean(sample_means)
```

```
3.48
```

```
# Estimate proportion of "Claire"s
np.mean(sample_props)
```

```
0.26
```



- Estimate characteristics of unknown underlying distribution
- More easily estimate characteristics of large populations

Let's practice!

INTRODUCTION TO STATISTICS IN PYTHON

The Poisson distribution

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Poisson processes

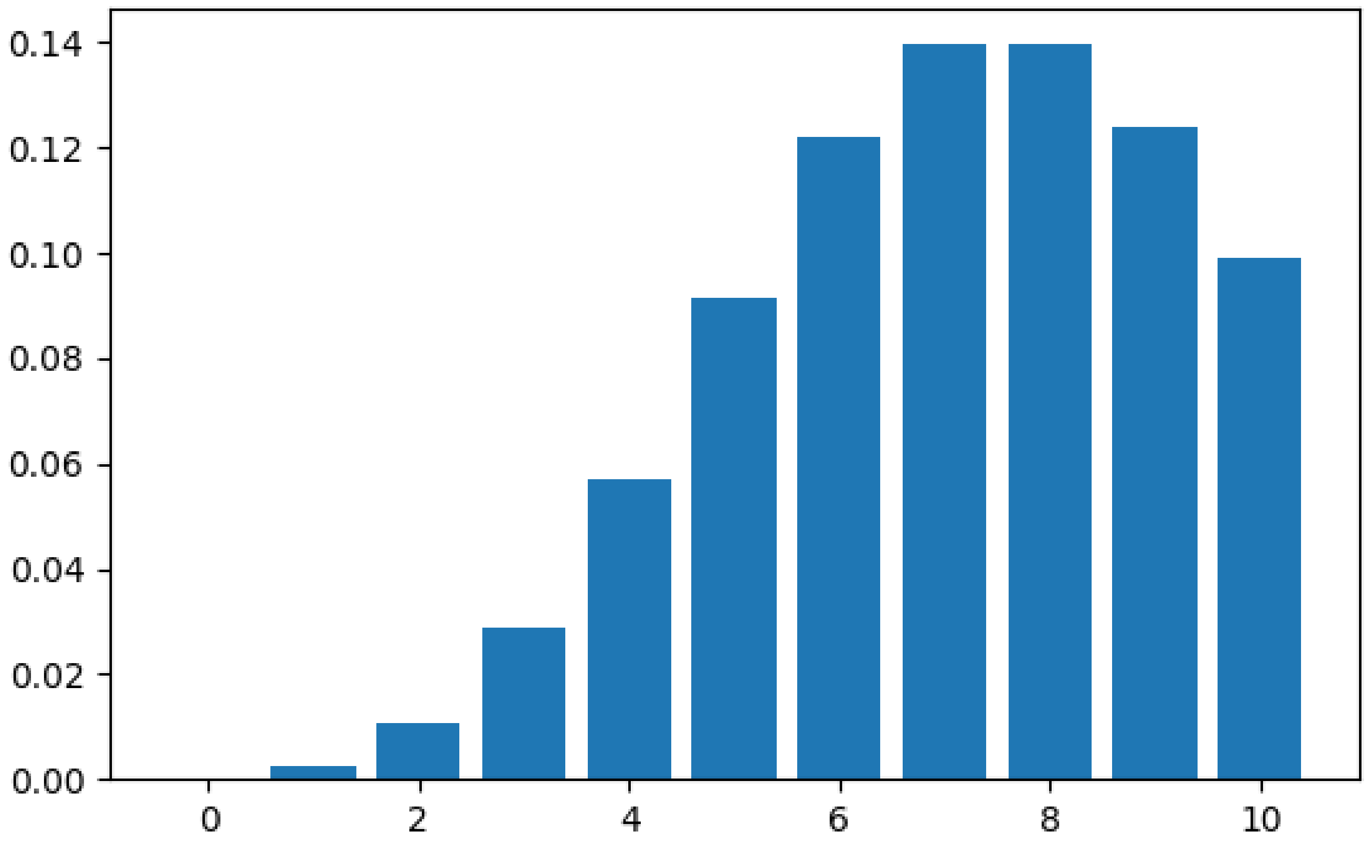
- Events appear to happen at a certain rate, but completely at random
- Examples
  - Number of animals adopted from an animal shelter per week
  - Number of people arriving at a restaurant per hour
  - Number of earthquakes in California per year
- Time unit is irrelevant, as long as you use the same unit when talking about the same situation

Poisson distribution

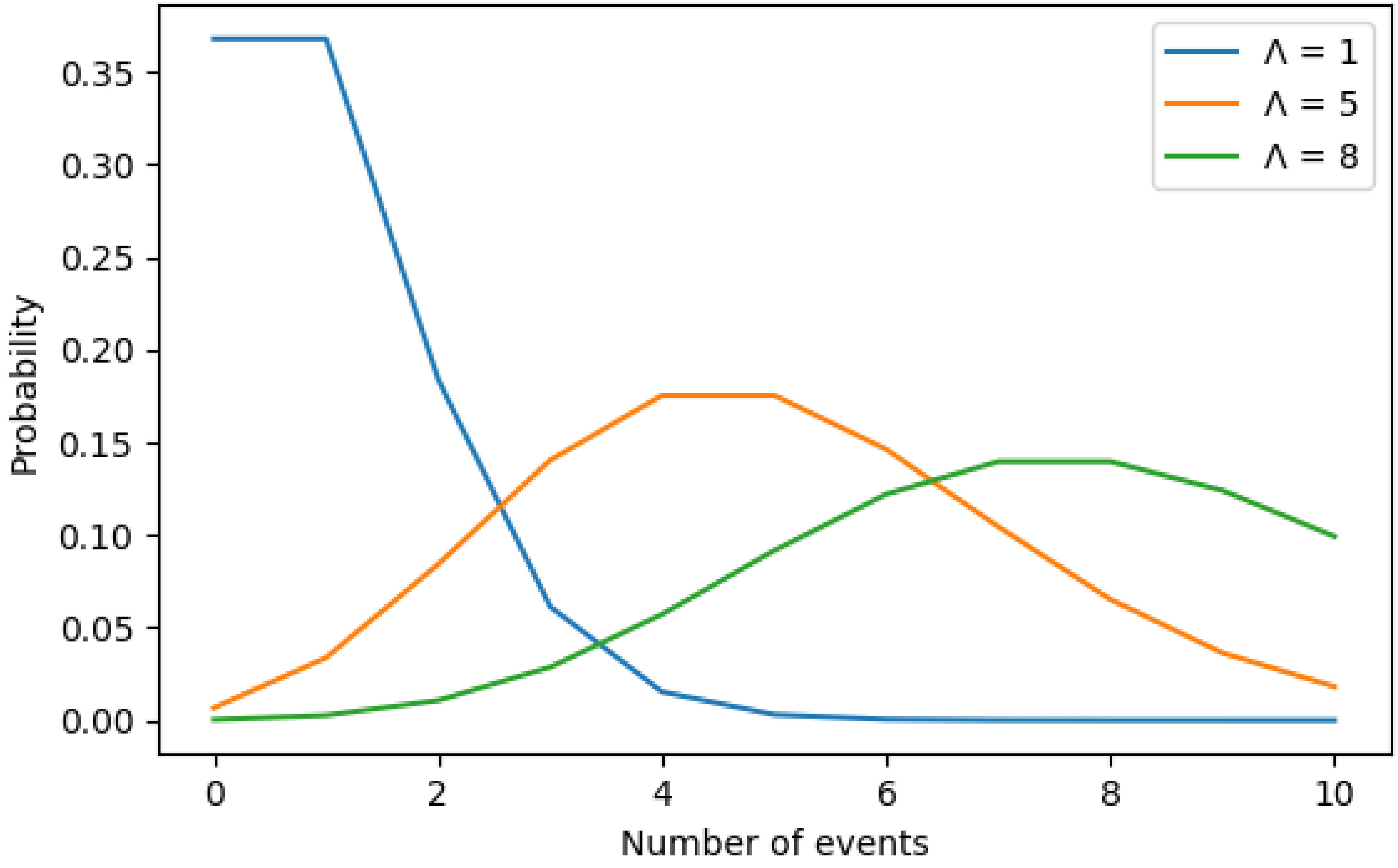
- Probability of some # of events occurring over a fixed period of time
- Examples
  - Probability of ≥ 5 animals adopted from an animal shelter per week
  - Probability of 12 people arriving at a restaurant per hour
  - Probability of < 20 earthquakes in California per year

Lambda (λ)

- λ = average number of events per time interval
  - Average number of adoptions per week = 8



Lambda is the distribution's peak



Probability of a single value

If the average number of adoptions per week is 8, what is $P(\#\text{ adoptions in a week} = 5)$?

```python
from scipy.stats import poisson
poisson.pmf(5, 8)
```

```
0.09160366
```

Probability of less than or equal to

If the average number of adoptions per week is 8, what is $P(\#\text{ adoptions in a week} \leq 5)$?

```
from scipy.stats import poisson
poisson.cdf(5, 8)
```

```
0.1912361
```

Probability of greater than

If the average number of adoptions per week is 8, what is $P(\# \text{ adoptions in a week} > 5)$?

```
1 - poisson.cdf(5, 8)
```

```
0.8087639
```

If the average number of adoptions per week is **10**, what is $P(\# \text{ adoptions in a week} > 5)$?

```
1 - poisson.cdf(5, 10)
```

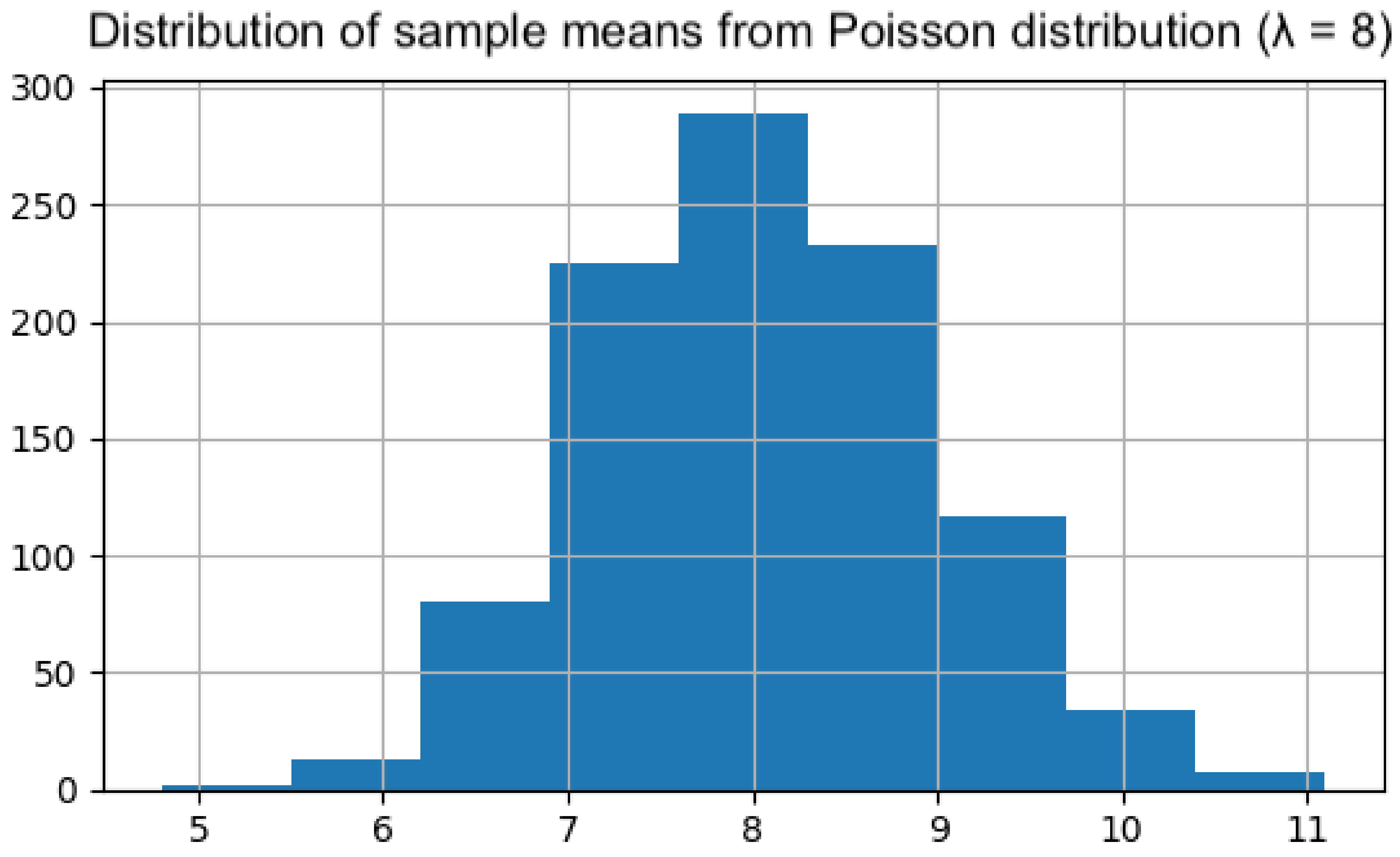
```
0.932914
```

Sampling from a Poisson distribution

```
from scipy.stats import poisson
poisson.rvs(8, size=10)
```

```
array([ 9,  9,  8,  7, 11,  3, 10,  6,  8, 14])
```

The CLT still applies!



Let's practice!

INTRODUCTION TO STATISTICS IN PYTHON

More probability distributions

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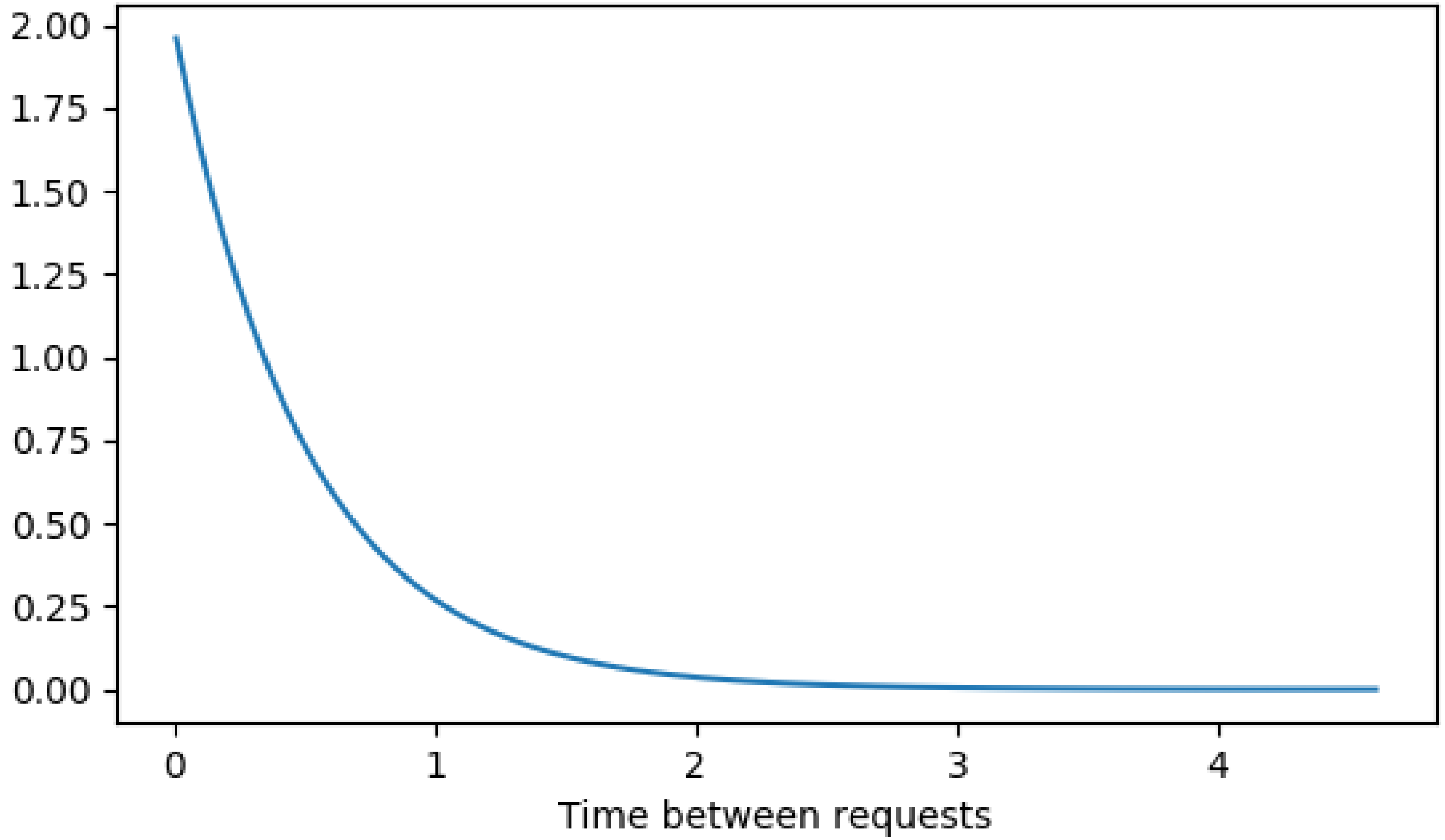
Exponential distribution

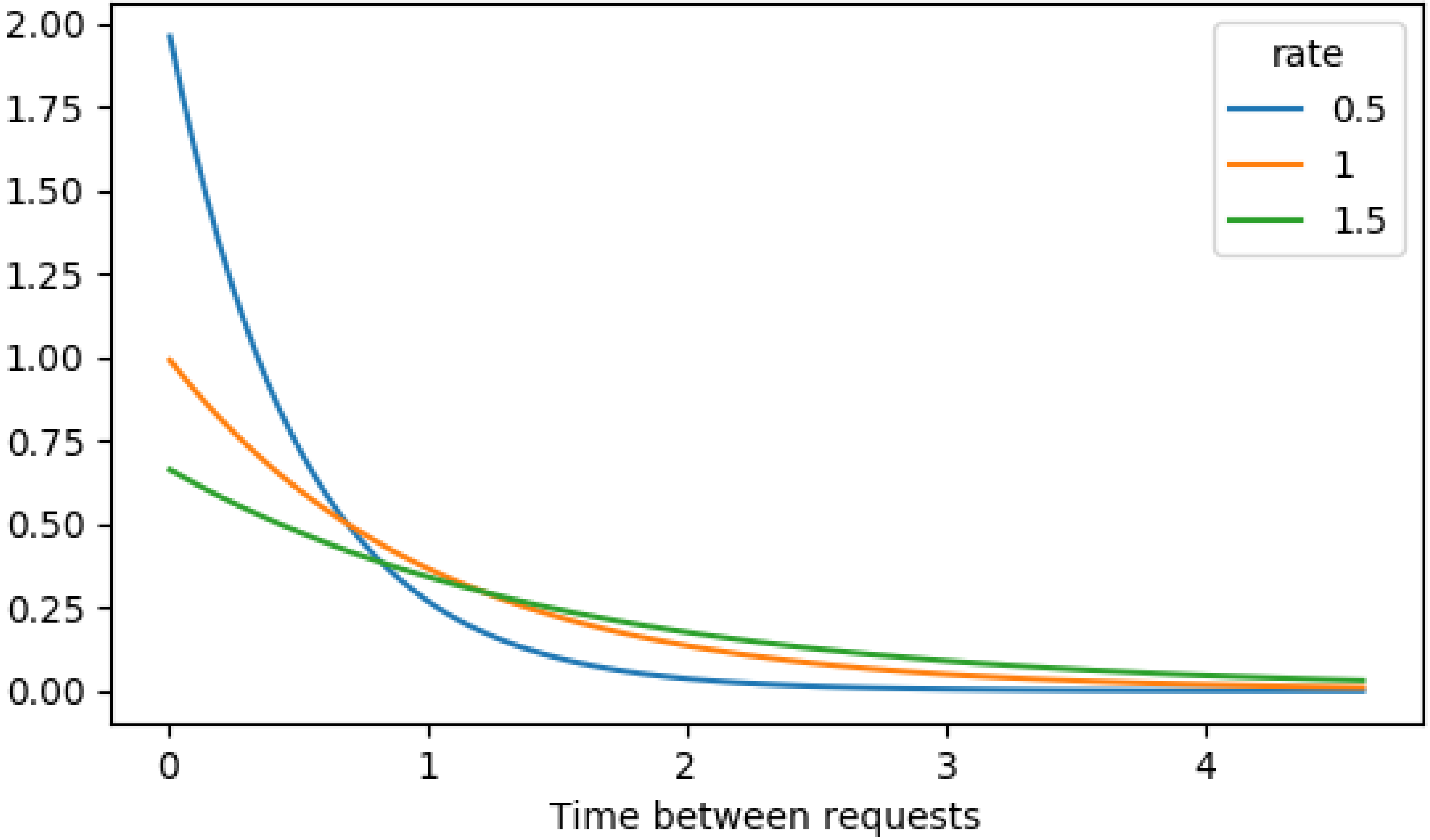
- Probability of time between Poisson events
- Examples
  - Probability of > 1 day between adoptions
  - Probability of < 10 minutes between restaurant arrivals
  - Probability of 6-8 months between earthquakes
- Also uses lambda (rate)
- Continuous (time)

Customer service requests

- On average, one customer service ticket is created every 2 minutes
  - λ = 0.5 customer service tickets created each minute



Lambda in exponential distribution



Expected value of exponential distribution

In terms of rate (Poisson):

- $\lambda = 0.5 \text{ requests}$ per minute

In terms of time between events (exponential):

- $1/\lambda$ = 1 request per 2 minutes
- $1/0.5 = 2$

How long until a new request is created?

```
from scipy.stats import expon
```

- `scale` $= 1/\lambda = 1/0.5 = 2$

$P(\text{wait} < 1 \text{ min}) =$

```
expon.cdf(1, scale=2)
```

```
0.3934693402873666
```

$P(\text{wait} > 4 \text{ min}) =$

```
1- expon.cdf(4, scale=2)
```

```
0.1353352832366127
```

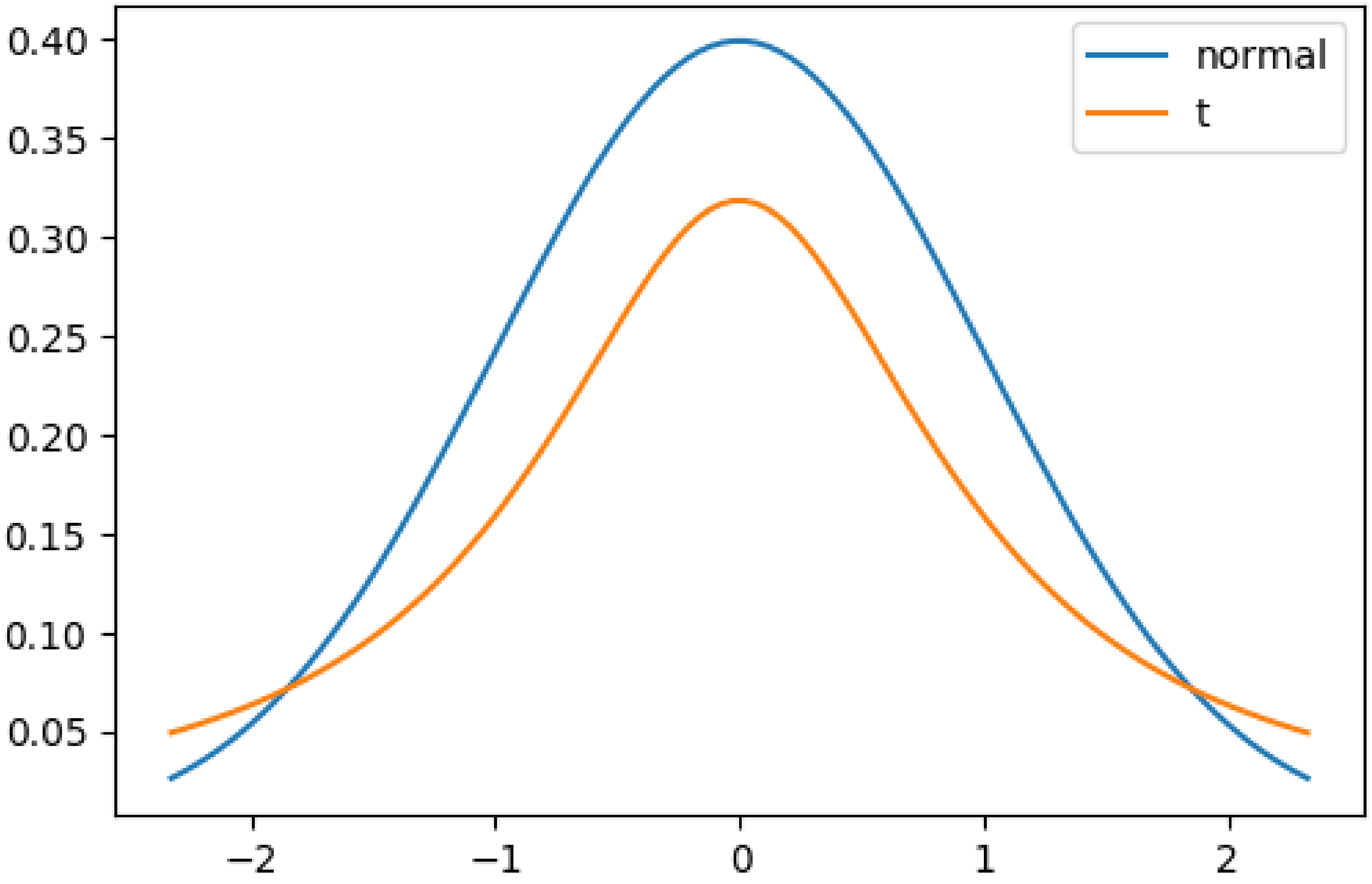
$P(1 \text{ min} < \text{wait} < 4 \text{ min}) =$

```
expon.cdf(4, scale=2) - expon.cdf(1, scale=2)
```

```
0.4711953764760207
```

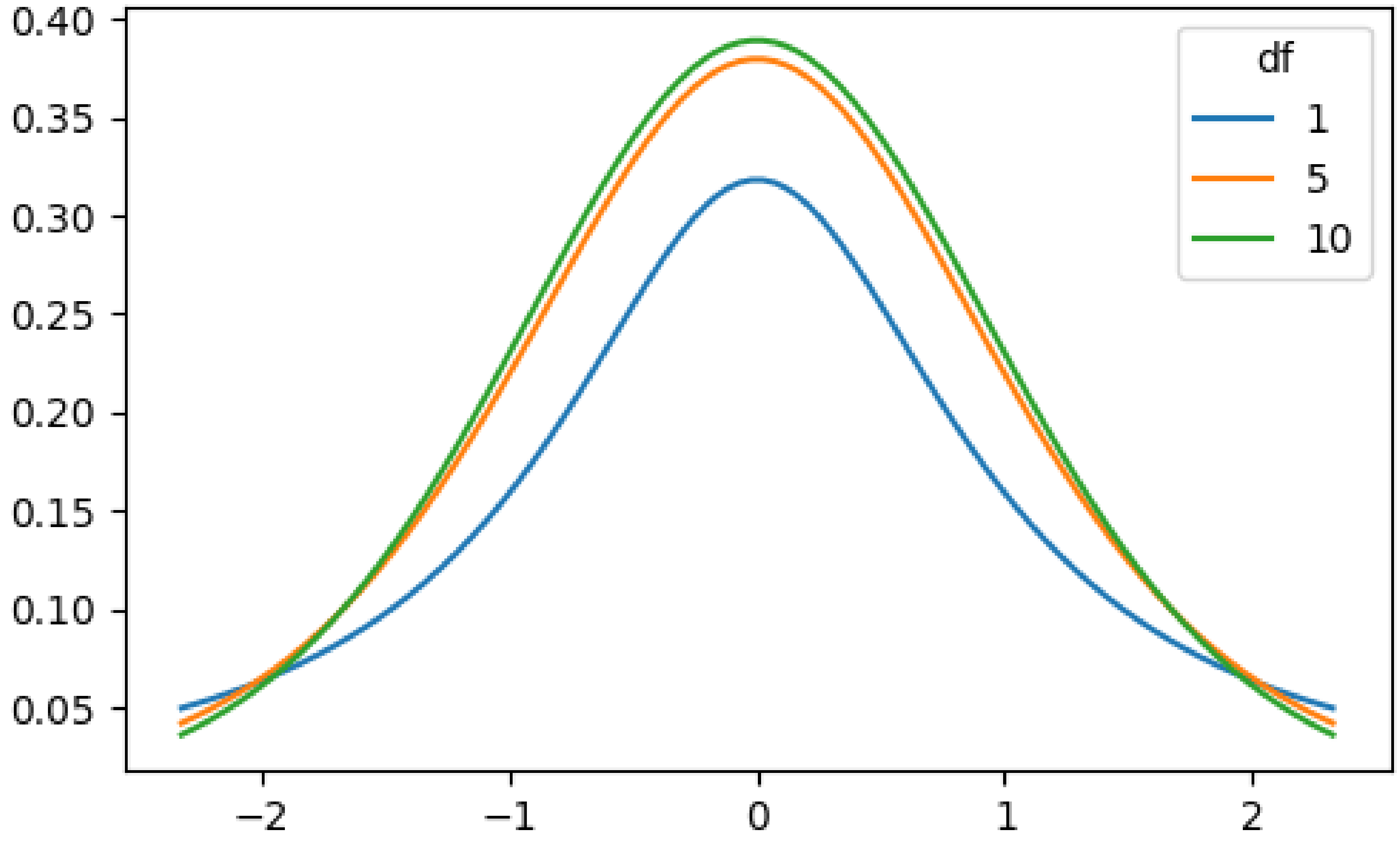
(Student's) t-distribution

- Similar shape as the normal distribution



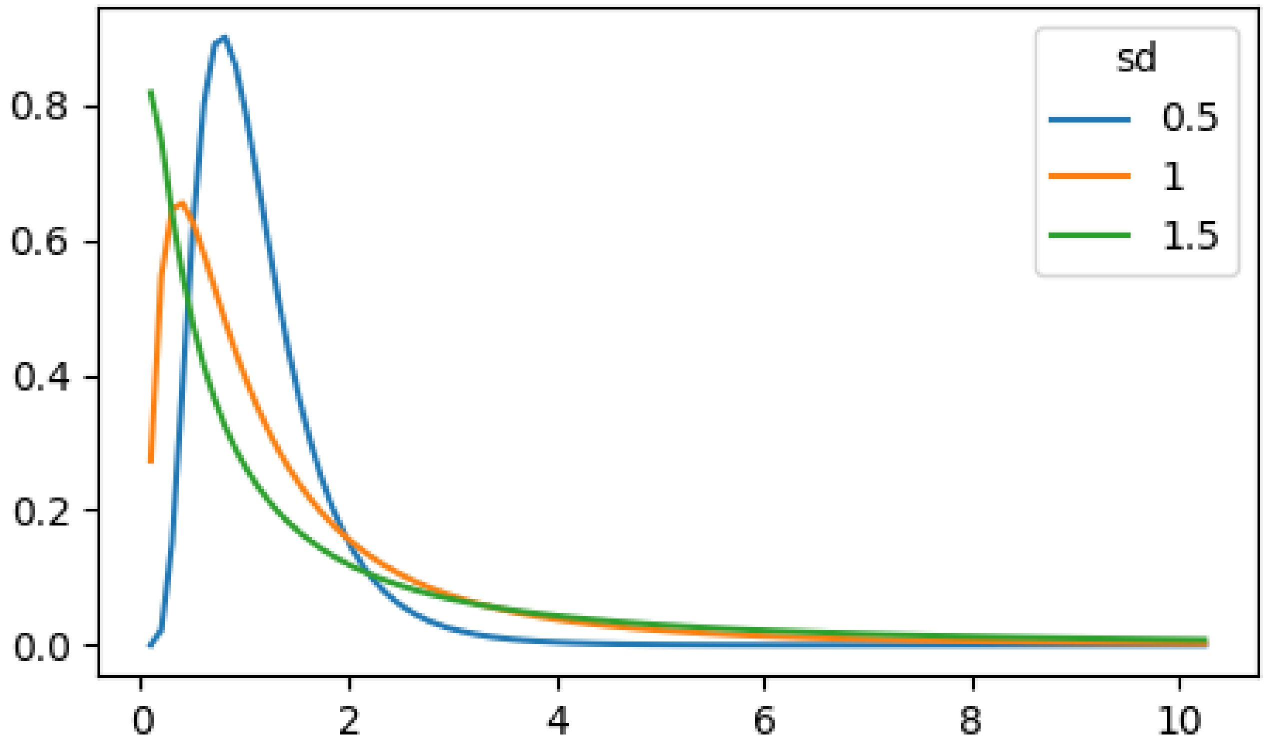
Degrees of freedom

- Has parameter degrees of freedom (df) which affects the thickness of the tails
  - Lower df = thicker tails, higher standard deviation
  - Higher df = closer to normal distribution



Log-normal distribution

- Variable whose logarithm is normally distributed
- Examples:
  - Length of chess games
  - Adult blood pressure
  - Number of hospitalizations in the 2003 SARS outbreak



Let's practice!

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